IPRO 307-Final Presentation

Title-

Designing A Three-Dimensional "Mesh" To Improve Quality Of Simulations For a Wide Variety of Professions

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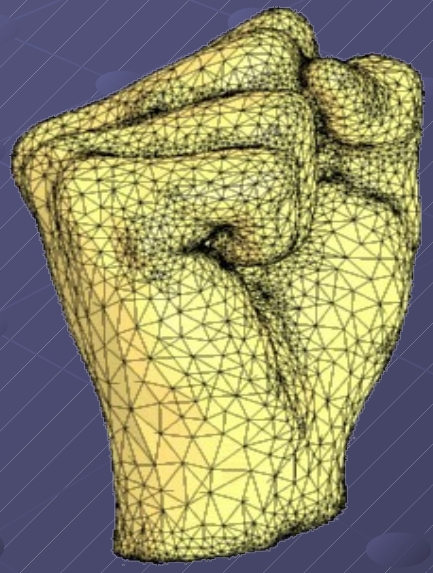

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Goal

To implement high quality automatic mesh generation program that generates high quality meshes using Delaunay refinement and sliver removal algorithms . At the same time, pay special attention on the robustness problem and efficiency of the program.

What are Meshes?

A Mesh is composed of triangles and tetrahedran which have a broad application area, such as computer graphic, scientific computation, numerical method computation, and etc.

Delaunay Triangulations

- *Delaunay triangulation* is a method of meshes that uses the technique of generating triangles and tetrahedrons.

Definition:

- *Delaunay triangulation – a geometric structure that is used to generate meshes. Triangles are used for 2-Dimensional meshes and tetrahedrons are used for 3-dimensional meshes.*

Properties of Delaunay Triangulation

If the circum-circle formed 2 points (A,B) does not enclose any other vertices or points, then the edge, AB, is delaunay.

Definition:

- *Circum-circle – A circle formed by edge of two connection points.*
- *Delaunay edge – The circum-circle of the edge of two vertices enclosed no other vertices .*

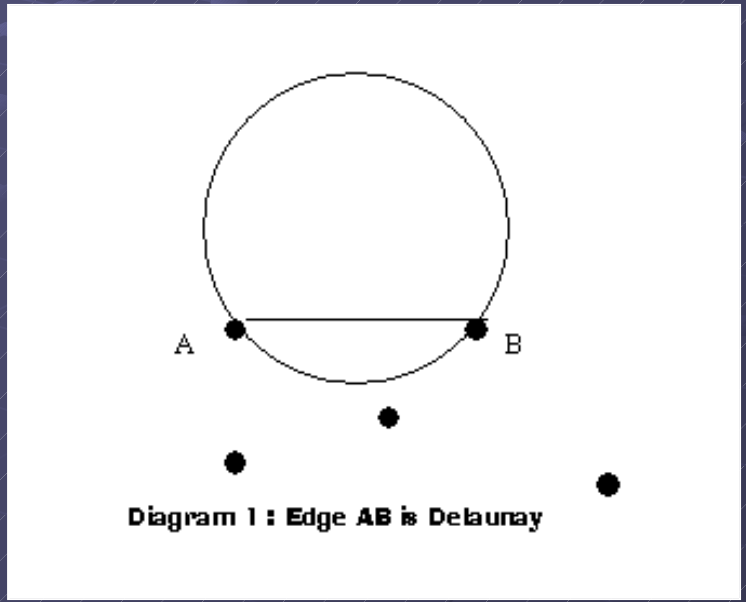


Diagram 1 : Edge AB is Delaunay

Properties of Delaunay Triangulation

If there is a triangulation T and the circumcircle of every triangle is empty, then all the edges of the triangles are Delaunay.

Definition:

- *Triangulation – A triangle formed by the vertices of three points.*
- *Delaunay Triangle – The circumcircle of the triangle enclosed no other vertices.*

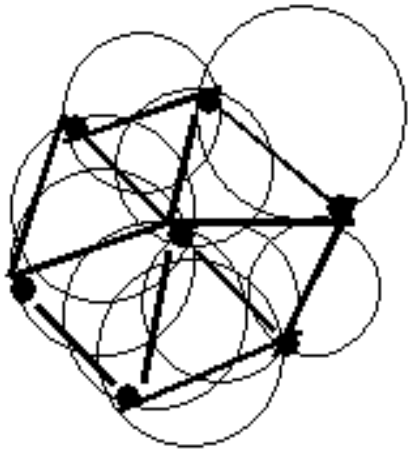


Diagram 2 : Delaunay Triangulation

Properties of Delaunay Triangulation

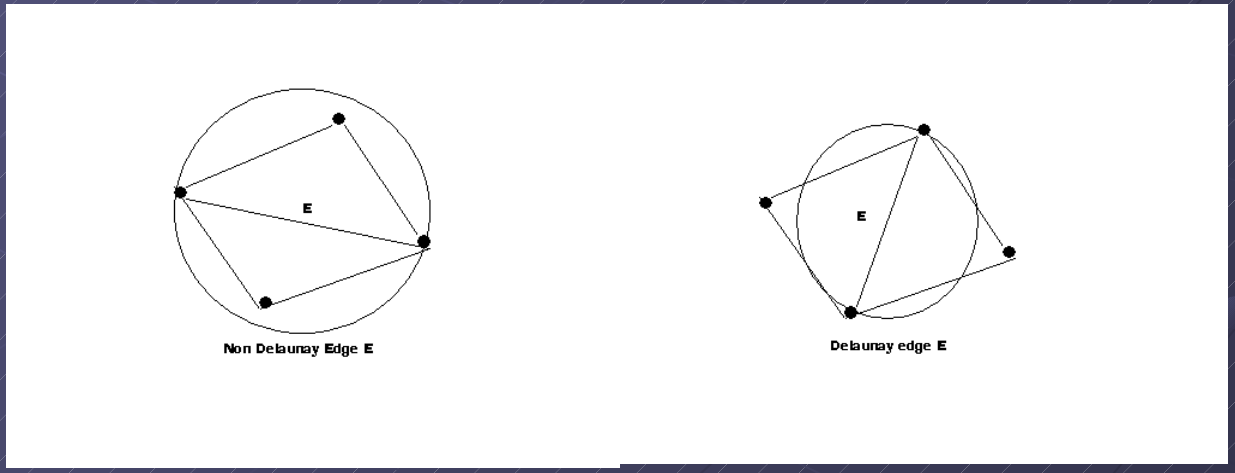
If the circum center of a triangulation encloses another vertices of points beside the three vertices that formed the triangulation, then the triangulation is not Delaunay.

Properties of Delaunay Triangulation

- If the edge E of the triangulation is not a Delaunay. The edge can be flipped and Delaunay achieved.

Definition:

- *Edge Flipping – When a triangulation is not Delaunay, The edge can be flipped within four vertices. It is mathematically proven that global Delaunay triangulation will be achieved with this method.*



Properties of Delaunay Triangulation

- If triangulation T is Delaunay,then all the edges are Delaunay. If the Triangulation is not Delaunay, one edge or more will not be Delaunay . The flip algorithm could proceed until all edges are Delaunay.

- A Flip algorithm can never become trapped in an endless loop.

Properties of Delaunay Triangulation

If given on a plane with the criteria that no more than three vertices are collinear and no four vertices are co-circular, Delaunay triangulation exists over the plane and the flip algorithm can be used to achieve complete Delaunay triangulation

Properties of Delaunay Triangulation

Delaunay triangulation,

- maximize the minimum angle in the triangulation
- minimize the largest circum-circle
- minimizes the largest min containment circle or the smallest circle containing a triangle.

- If the optimal triangulation could not be improved, then, Delaunay triangulation is achieved.

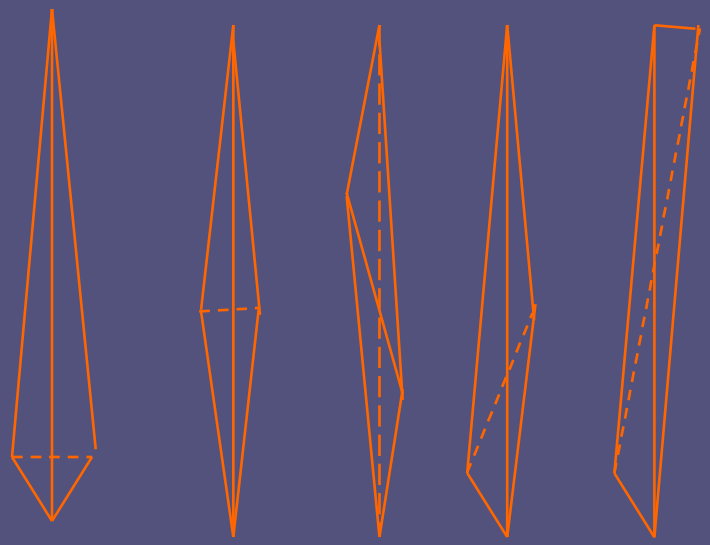
Delaunay Tetrahedralization

Definition:

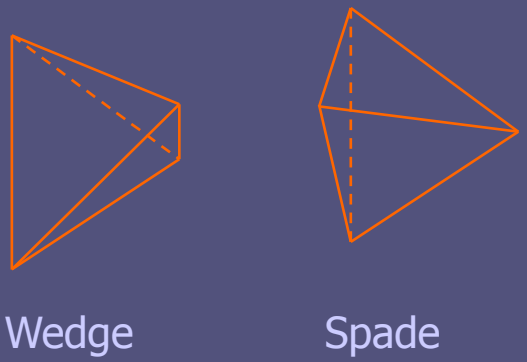
- *Delaunay Tetrahedralization* – Tetrahedrons are generated in this method of meshes.(Tetrahedrons – A Triangle base pyramid).

- Delaunay Tetrahedralization is not as efficient as Delaunay triangulation but it posed as an important instrument in three-dimensional meshes.

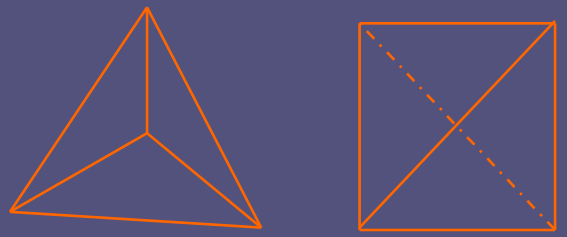
Badly Shaped Tetrahedrons



Spire Spear Spindle Spike Splinter



Wedge Spade



Cap Sliver

Delaunay Refinement

- It is a successful algorithm that has been formulated to remove the sliver and bad sectors, hence, creates a good quality 3-dimensional tetrahedralization.

Slivers

Sliver – A tetrahedron whose four vertices lie close to a plane and whose perpendicular projection to that plane is a convex quadrilateral with no short edge.

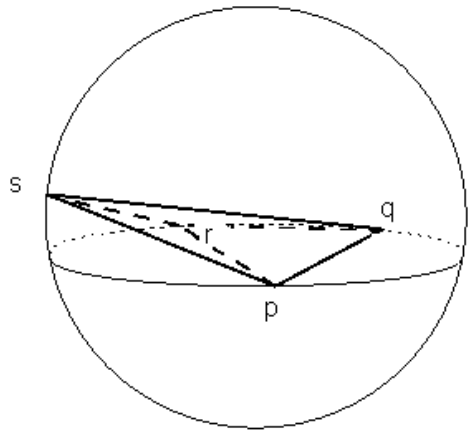


Diagram 1 : Sliver

Refinement Algorithm

- In order to determine whether a tetrahedron is a bad tetrahedron or sliver, it would have to meet some criteria. A sliver fulfills the first criteria RT/ LT < ρ and second criteria, V/ LT3 ≤ σ.

- Refinement algorithm starts with checking with the criteria, RT/ LT > ρ .

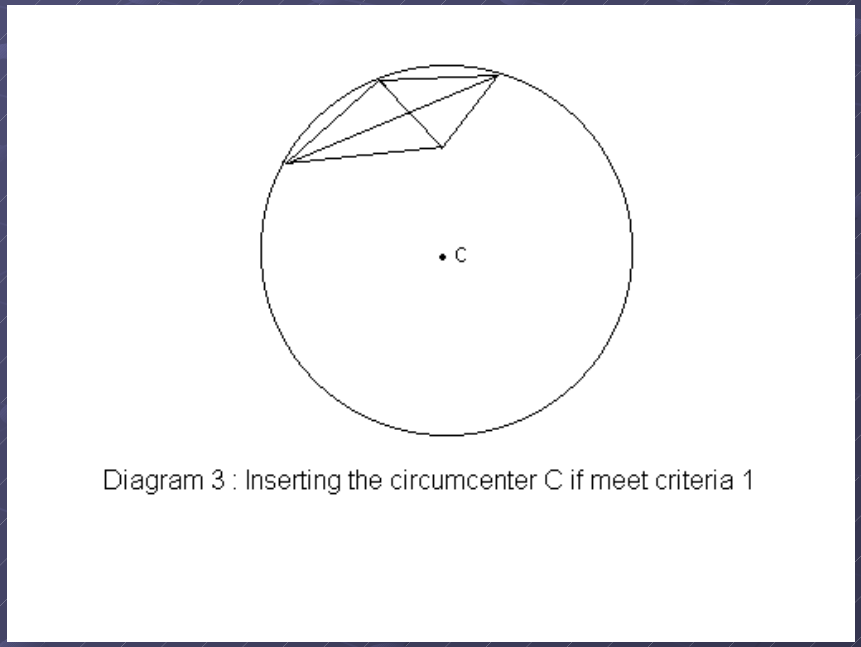


Diagram 3 : Inserting the circumcenter C if meet criteria 1

Refinement Algorithm

- We also use the criteria V/ LT3 for testing. If V/ LT3 > σ, the particular tetrahedron is good, but, if V/ LT3 ≤ σ, then the tetrahedron is the silver. Then we randomly choose point P that satisfies |p-c| ≤ δ·RT . . The new tetrahedrons have small RT/ LT Ratio and the circumradius R of a tetrahedron is R ≤ b·RT where b is some constant.

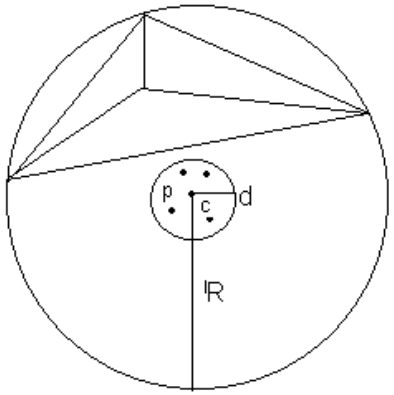


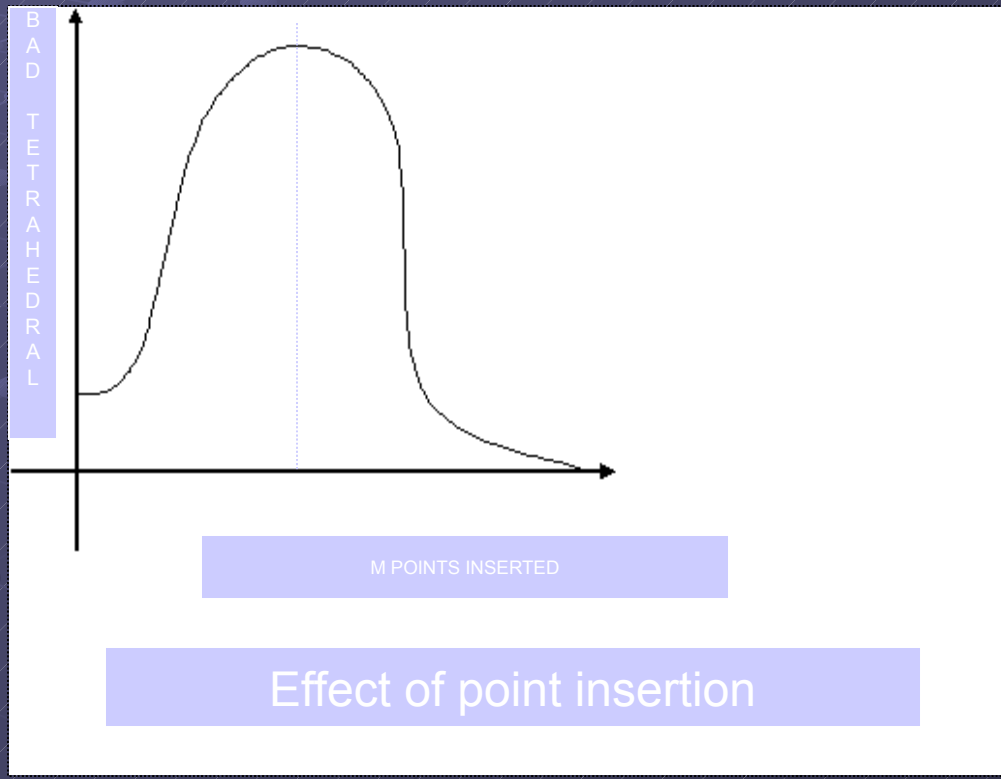
Diagram 4 : Point insertion if the case is a sliver

Refinement Algorithm

- The same process will be repeated for the N number of points. From the N points that we generated, we need to determine best point to insert.

  Qp = MINTP VTP/L TP 3 is the formula to determine the quality of a particular point.

  When a new point is inserted, the particular bad tetrahedron will be removed and replaced with new tetrahedrons.



Effect of point insertion

Mesh Data Structure

- *Definition:*

  Mesh Data Structure – The representation of tetrahedron mesh in term of structured data object .

  To ensure the data structure was flexible and reliable, three different relative data structures are used-
- *Tetrahedron-based Data Structure* – It is a record to represent each tetrahedral in the mesh. Each record contains four pointers to its vertices, fours pointers to adjoining tetrahedra, four pointers to subsfaces, and one optional user-defined attribute .
- *Shellface* – It is a data structure that used to represent subfacet and segment. Subfacet contains three pointers to its vertices, three pointers to adjoining tetrehedra, and one boundary marker. Segment contains only a pointer to one adjoining subfacet.
- *Point3d* – It simply represents a point in 3D dimension and its properties.