Stochastic mechanism of color confinement

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It is shown that in stochastic QCD a vacuum color quark is confined due to the interaction with environment, chaotic dynamics of Yang-Mills-Higgsfields, decoherence of pure color state into mixed white (colorless) state and also squeezed, and entangled states appearance. Critical energy of order-chaos transition is obtained which depends on Higgs boson mass. Stochasticity is the root of color confinement disappearing of color at confinement radius.

1. Stochasticity

Let us take a heavy spinless color particle ("quark") in the QCD vacuum, for example inside a hadron or deconfined QGP. QCD vacuum is the environment for color quantum particles whose properties are averaging over all external QCD vacuum implementations [1–5]. Interactions with the environment result in decoherence and relaxation of quantum superpositions [6, 7]. Interactions of some quantum system with the environment can be effectively represented by additional stochastic terms in the Hamiltonian of the system. QCD vacuum represents itself namely as a stochastic (not coherent one) system. Stochastic means that only the second order correlators in the QCD vacuum are dominated (Gauss domination). It has been confirmed by lattice calculation. The most important evidence for this is Casimir scaling [8]. The model of the QCD stochastic vacuum is one of the popular phenomenological models which exhibits quark confinement, string tension and field configurations around static charges [9–12]. When considering a QCD stochastic vacuum as the environment for color quantum particles with the averaging over external QCD stochastic vacuum implementations, we obtain as consequences decoherence, relaxation of quantum superpositions, loss of information, and confinement of color states.

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2. Density matrix

In the situation of a quantum system (quark) in the environment (stochastic QCD vacuum) a density matrix is the most adequate formalism. The color particle density matrix of the system taking into account both color particle and QCD stochastic vacuum as environment is obtained by averaging with respect to the stochastic terms [1–5]

$$\rho(\mathrm{loop}, 1\ 2) =< |\phi(1)\rangle\langle\phi(2)| >, \tag{1}$$

here we average over all implementations of stochastic gauge field (environment degrees of freedom). In the model of QCD stochastic vacuum only expectation values of path ordered exponents over closed paths are defined. The amplitude is obtained by parallel transport [1–5]

$$|\phi(1)\rangle = \mathcal{P}\exp\left(i\int dx^{\mu}\hat{A}_{\mu}\right)|\phi_{in}\rangle, \tag{2}$$

Closed path corresponds to a process in which the particle-antiparticle pair is created, propagates and finally annihilates. With the help of (1) and (2) we can obtain the next expression

$$\rho(\mathrm{loop}, 1\ 2) = N_c^{-1} + (|\phi_{in}\rangle\langle\phi_{in}| - N_c^{-1})W_{adj}(\mathrm{loop}, 1\ 2). \tag{3}$$

Here N_c is a number of colors, $W_{adj}(\mathrm{loop}, 1\ 2)$ is the Wilson loop in the adjoint representation, and we have used the property that color density matrix in color neutral stochastic vacuum can be decomposed into the pieces transformed under trivial and adjoint representations [4].

As is known due to Casimir scaling, decay rates of Wilson loops in different representations are proportional to each other , in particular $W_{fund}(\mathrm{loop}, 1\ 2)$ and $W_{adj}(\mathrm{loop}, 1\ 2)$. Decay of $W_{fund}(\mathrm{loop}, 1\ 2)$ points at confinement of color charges. Simultaneously we have decay of $W_{adj}(\mathrm{loop}, 1\ 2)$ that means from Eq. (3) that the color density matrix obtained as a result of parallel transport along the (loop, 1 2) tends under the confinement regime to the color density matrix of white (colorless) mixture $\rho = N_c^{-1}$. Here all color states are mixed with equal probabilities and all information on initial color state is lost. The stronger the color states are confined the stronger their states transform into the white mixture.

So, as the Wilson area law holds for the Wilson loop (confinement criterion), we can obtain an explicit expression for the density matrix if we choose for example the rectangular (loop 1 2) spanned in terms of time T and distance R [2, 4]. When R or T are of order of 1 fm (for SU(3) theory), the Wilson loop decays exponentially with the area spanned on (loop 1 2)

$$\rho(\mathrm{loop}, 1\ 2) = N_c^{-1} + (\rho_{in} - N_c^{-1})\exp(-\sigma_{adj}RT), \tag{4}$$

where $\sigma_{adj} = \sigma_{fund} G_{adj} G_{fund}^{-1}$ is string tension in the adjoint representation and G_{adj}, G_{fund} are the eigenvalues of quadratic Casimir operators. Under the condition of Gaussian dominance, string tension is $\sigma_{fund} = g^2 l_{corr}^2 F^2/2$, where g is the coupling constant, l_{corr} is the correlation length in the QCD stochastic vacuum, and F^2 is the average of the second cumulant of curvature tensor when $g^2 l_{corr}^2 F^2 \ll 1$ [12].

The decoherence rate of transition from pure color states to white mixture can be estimated on the base of purity [6] $P = \mathrm{Tr}\ \rho^2$ [4]

$$P = N_c^{-1} + (1 - N_c^{-1}) \exp(-2\sigma_{fund} G_{adj} G_{fund}^{-1} RT). \tag{5}$$

When T or R tend to 0, $P \to 1$, that corresponds to pure state with the density matrix $\rho_{in} = |\phi_{in}\rangle\langle\phi_{in}|$. When composition RT tends to infinity the purity tends to N_c^{-1}, that corresponds to the white mixture state with the density matrix N_c^{-1}. The rate of purity decreasing is $T_{dec}^{-1} = 2\sigma_{fund} G_{adj} G_{fund}^{-1}$, where T_{dec} is the characteristic time of decoherence proportional to QCD string tension and distance R. It can inferred from (3) and (4) that the stronger is particle-antiparticle pair coupled by QCD string or the larger is the distance between particle and antiparticle the quicker information about color state is lost as a result of interaction with the QCD stochastic vacuum. Thus white states can be obtained as a result of decoherence process which allows to conjecture analogy with color particle confinement. Information on quark color states in confinement region is lost due to interactions between quarks and confining non-Abelian gauge fields (stochastic QCD vacuum).

3. Confinement, fidelity, critical energy of order-chaos transition and mass of the Higgs boson

The Wilson loop definition in QCD is similar [13] to the definition of fidelity [8], the quantity which describes the stability of quantum motion of the particles [14]. Using the analogy between the theory of gauge fields and the theory of holonomic quantum computations [13, 15, 16] we can define the fidelity of quark motion. We consider the motion of color particles in different paths from the point x to the point y. In the initial point x state vectors are $|\phi_{in}\rangle$. For large particle mass and taking into account that because of Hermitian character of $\hat{A}_\mu$ operator (1) is unitary. We can rewrite fidelity as integral over the closed loop, traveling from point x to the point y

$$f = < \langle\phi_{in}| \mathcal{P} \exp\left(i \int dx^\mu \hat{A}_\mu \right) |\phi_{in}\rangle > \tag{6}$$

in the path 1 and back to the point x in the path 2 and obtain integral proportional to the identity due to the color neutrality of stochastic vacuum.

The final expression for the fidelity of the particle moving in the Gaussian-dominated stochastic vacuum is

$$f = \exp\left(-\frac{1}{2} g^2 l_{corr}^2 F^2 S\right), \tag{7}$$

where S is the area of the surface spanned over the contour $(\mathrm{loop}, 1\ 2)$. Thus the fidelity for color particle moving along contour decays exponentially with the surface spanned over the contour S the decay rate being equal to the string tension is $\sigma_{fund} = g^2 l_{corr}^2 F^2/2$. Another situation, more close to the standard treatment of the fidelity, is realised when 1 and 2 are two random paths in Minkowski space, closed to each other. The corresponding expression for the fidelity is similar, but now the averaging is performed with respect to all random paths which are close enough. If the unperturbed path is parallel to the time axis in Minkowski space, the particle moves randomly around some point in three dimensional space. The fidelity in this case also decays exponentially with time. Thus we have close connection between confinement and instability of color particle motion.

The increasing of instability of motion in the confinement region is also connected with existence of chaotic solutions of Yang-Mills field [1, 17], possible chaos onset [18]. Yang-Mills fields already on classical level show inherent chaotic dynamics and have chaotic solutions [17, 18]. It was shown that the Higgs boson and its vacuum quantum fluctuations regularize the system and lead to the emergence of order-chaos transition at some critical energy [13, 19–21]

$$E_c = \frac{3\mu^4}{64\pi^2} \exp(1 - \frac{\lambda}{g^4}). \tag{8}$$

Here μ is mass of Higgs boson, λ is its self-interaction coupling constant, g is the constant coupling gauge and Higgs fields. Very important here is the value of mass of Higgs boson. From Ref. [22]: "Higgs mass lower than some critical value and potential is unstable, and the universe can phase transition to another vacuum." On the other hand, in the region of confinement there exists the boundary of order-chaos transition where the fidelity decreases exponentially and which is equal to string tension $\sigma_{fund} = g^2 l_{corr}^2 F^2/2$. This connects the properties of stochastic QCD vacuum, Higgs boson mass and coupling constants.

4. Squeezed and entangled color states

The instability of motion in the confinement region is also connected with possible phenomena of quantum entanglement and squeezing of color states [23–27].

Color particles moving through QCD vacuum with large momentum transfered develop quark-gluon jets. Both perturbative and nonperturbative (with sub-Poissonian multiplicity distributions) stages of the jet evolution are important [28]. Gluon multiplicity distribution at the end of the perturbative cascade in the range of the small transverse momenta (thin ring of jet) is Poissonian one [29]. Multiplicity distribution for the whole jet at the end of the perturbative cascade can be represented as a combination of Poissonian distributions (coherent states.). Gluon coherent states under the influence of the nonlinearities of QCD Hamiltonian transform into the squeezed and entangled states with sub-(super-)Poissonian multiplicity distributions [24–26]. Within local parton-hadron duality we can estimate nonperturbative contribution of the gluon squeezed states to the pion correlation functions in the jet narrow ring [30].

The emergence of entangled and squeezed states in QCD becomes possible due to the four-gluon self-interaction, the three-gluon self-interaction does not lead to the effects [24–26]. In principle, these effects are possible even for quadratic Hamiltonians in the quantum theory under certain conditions. Moreover we may amplify or, on the contrary, weaken both the squeezing effect and the system instability [31].

Two mode gluon squeezed and entangled states with two different colors can lead to quark-antiquark entangled states, the role of which could be important for of the confinement and hadronization phenomena [25, 26].

Quantum entanglement for cubits and Yang-Mills-Higgs fields was considered in [27] in terms of the original quasiclassical formalism developed in [31]. The concept of quantum entanglement was found to be very useful as a model-independent characteristic of the structure of the ground state of quantum field theories which exhibit strong long range correlations, most notably lattice spin systems near the critical points and the corresponding conformal field theories [32].

Quantum entanglement was also considered as an alternative way to probe the confining properties of large-N gauge theories [33, 34]. Quantum entanglement between the states of static quarks in the vacuum of pure Yang-Mills theory was analyzed in Ref. [33].

The Hilbert space of physical states of the fields and the charges is endowed with a direct product structure by attaching an infinite Dirac string to each charge. Tracing out the gauge degrees of freedom yields the density matrix which depends on the ratio of Polyakov and Wilson loops spanned on quark world lines. In the confinement regime, the entanglement of quark color states is maximal [35].

5. Conclusions

We have shown that a stochastic (not coherent) vacuum of quantum chromodynamics for which only correlators of the second order are important can be considered as the environment (in the sense of quantum optics) for color particles (quarks and gluons), where the Wilson loop corresponds to the fidelity of quantum color particle motion and confinement to the instability (chaoticity) of motion and to decoherence of pure color states into mixed white states. The Wilson loop, fidelity and purity decay exponentially with decay rate equal to the string tension. The dynamics of Yang-Mills fields, which is inherently chaotic one already at the classical level, can be partly regularized by interaction with Higgs fields and by quantum fields fluctuations. The critical point of an order-chaos transition appears which corresponds to the point of fidelity exponential decreasing. Squeezinq, entanglement, decoherence and instability accompany nonperturbative evolution of colour particles in QCD vacuum and confinement phenomenon.

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