Measurement of anisotropic radial flow rapidity

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Using the sample of Au + Au collisions at 200 GeV generated by the AMPT with string melting model, the anisotropic amplitudes of azimuthal distributions of total transverse momentum, mean radial (transverse) momentum, and multiplicity are first presented and compared. It shows that the azimuthal distribution of mean radial momentum well characterizes the radial expansion. So a measurement of the azimuthal distribution of mean transverse (radial) rapidity of final state particles is suggested. We further show that the isotropic part of the suggested distribution is the combination of isotropic radial expansion and thermal motion. The anisotropic amplitude characterizes the anisotropic radial flow, and coincides with the parameter of anisotropic radial flow rapidity extracted from a generalized blast-wave parametrization.

1. Introduction

The observation of large elliptic flow at RHIC is considered as one of the most important signatures of the formation of the strongly interacting Quark Gluon Plasma (sQGP) [1, 2]. The flow harmonics are Fourier coefficients of the azimuthal multiplicity distribution of final state hadrons [3]. One common feature of flow harmonics is their mass ordering in the low transverse momentum region [4]. This phenomena can be well understood by hydrodynamics with a set of kinetic freeze-out constraints, i.e., the temperature, the radial flow, and the source deformation [5]. The radial flow is usually described by 2 parameters. The first is the isotropic radial velocity (or rapidity, related by $v_{\mathrm{T}} = \tanh y_{\mathrm{T}}$). It presents the surface profile of isotropic transverse expansion of the source at kinetic freeze-out.

The other parameter is the anisotropic radial velocity (i.e., the azimuthal dependent radial velocity). It measures the difference of the radial flow strength in and out of the reaction plane. It is introduced to account for

the anisotropic radial flow field which arises in *non-central* collisions. The observed elliptic flow can be generated by anisotropic radial flow [5, 6]. Moreover, the shear tension of viscosity in hydrodynamics is supposed to be proportional to the gradient of radial velocity along the azimuthal direction [7], which is directly related to anisotropic radial velocity. The proportionality constant is the shear viscosity.

In hydrodynamic models [8, 9, 10], these parameters are not independent. They are related by the initial conditions and the equation of state. Their determination is crucial for theoretical calculations.

It has been shown that the azimuthal distribution of mean transverse momentum directly measures the transverse motion of the source at kinetic freeze-out [11]. So we suggest the measurement of the azimuthal distribution of the mean transverse rapidity of final state hadrons ($\langle y_{\mathrm{T}}(\phi) \rangle$). It should be helpful in determining the parameters of the anisotropic radial rapidity.

As we know, $\langle y_{\mathrm{T}}(\phi) \rangle$ contains three parts: average isotropic radial rapidity, average anisotropic radial rapidity, and average thermal motion rapidity [4]. Since both thermal and radial motions contribute to the isotropic rapidity of the distribution, the isotropic radial rapidity itself can not be directly obtained from the distribution. Conventionally, the radial flow parameters are extracted from the p_{T} spectra of the hadrons [12, 13], or dileptons [14, 15], by a generalized blast-wave parametrization [5, 6]. The obtained parameters are an approximate description of the radial flow; they are model dependent. A model independent measure is called for.

Fortunately, the thermal motion is isotropic. As such, it does not contribute to the anisotropic radial flow. The azimuthal amplitude of the mean transverse rapidity distribution should correspond directly to the anisotropic radial rapidity. It is interesting to see the features of the azimuthal distribution of mean transverse rapidity, and how its azimuthal amplitude relates to the parameters of anisotropic radial rapidity extracted by a generalized blast-wave parametrization.

In this paper, using a sample generated by the AMPT model with string melting [16, 17], we first compare the anisotropic amplitudes of three azimuthal distributions of total transverse momentum, the mean radial (transverse) momentum, and multiplicity in section II. It shows that the azimuthal distribution of mean radial momentum, or rapidity, is a good measure of radial expansion. In section III, we further demonstrate the measured physics of isotropic and anisotropic amplitudes of the suggested distribution. They behave indeed as the expected radial flow (with a random thermal component), and anisotropic radial flow, respectively. In section IV, it is further shown that the parameter of anisotropic radial flow rapidity is just the anisotropic amplitude of the suggested distribution. Finally, the summary and conclusions are given in section V.

2. Measurements of radial expansion

Conventionally, the azimuthal distribution of the multiplicity of final state particles is presented by

$$\frac{dN}{d\phi} \propto 1 + \sum_{n=1}^{\infty} 2v_n(N)\cos(n\phi), \tag{1}$$

where ϕ is the azimuthal angle between the transverse momentum of the particle and the reaction plane. The coefficients of the Fourier expansion are [18],

$$v_n(N) = \langle \cos(n\phi) \rangle, \tag{2}$$

where $\langle \ldots \rangle$ is an average over all particles in all events. The second harmonic coefficient $v_2(N)$ is the so-called elliptic flow parameter. It represents the anisotropy of the colliding system and has the biggest value in relativistic heavy ion collisions [19].

However, the multiplicity distribution only counts the number of particle emissions in a certain azimuthal angle. The initial anisotropy in coordinate space in non-central collisions makes the formed system expand in a perpendicular almond shape in momentum space. The expansion of the system generates not only the anisotropy of multiplicity distribution but also their associate radial (transverse) momentum. The total radial momentum at a given azimuthal angle is the combination of them. Therefore, the azimuthal distribution of radial momentum is a good measure of the anisotropic expansion. The total transverse momentum in the mth azimuthal bin can be defined as

$$\langle P_t(\phi_m) \rangle = \frac{1}{N_{\text{event}}} \sum_{j=1}^{N_{\text{event}}} \left(\sum_{i=1}^{N_m} p_{T,i}(\phi_m) \right). \tag{3}$$

Where $p_{T,i}$ is the transverse momentum of the ith particle, N_m is the total number of particles, and $\langle \ldots \rangle$ denotes the average over all events.

In order to see the contributions of radial expansion alone, the mean radial momentum in the mth azimuthal bin can be defined accordingly as,

$$\langle \langle p_T(\phi_m) \rangle \rangle = \frac{1}{N_{\text{event}}} \sum_{j=1}^{N_{\text{event}}} \left(\frac{1}{N_m} \sum_{i=1}^{N_m} p_{T,i}(\phi_m) \right). \tag{4}$$

Here, the averages $\langle \langle \ldots \rangle \rangle$ are over all particles in the mth angle bin and all events. This records only the contributions from the transverse momentum of final particles. In contrast to the azimuthal multiplicity distribution, the multiplicity effect is eliminated by the average over the number of particles in the mth bin.

The anisotropic parameters of all those azimuthal distributions can be directly obtained from their Fourier expansions,

$$\frac{d\langle P_{\mathrm{t}}\rangle}{d\phi} \propto 1 + \sum_{n=1}^{\infty} 2v_n(\langle P_{\mathrm{t}}\rangle)\cos(n\phi), \tag{5}$$

and

$$\frac{d\langle\langle p_{\mathrm{T}}\rangle\rangle}{d\phi} \propto 1 + \sum_{n=1}^{\infty} 2v_n(\langle\langle p_{\mathrm{T}}\rangle\rangle)\cos(n\phi). \tag{6}$$

$\frac{d\langle P_{\mathrm{t}}\rangle}{d\phi}$ and $\frac{d\langle\langle p_{\mathrm{T}}\rangle\rangle}{d\phi}$ are the azimuthal distribution functions of total radial momentum and mean radial momentum. $v_n(\langle P_{\mathrm{t}}\rangle)$ and $v_n(\langle\langle p_{\mathrm{T}}\rangle\rangle)$ are their anisotropic parameters, respectively.

In order to see the contributions of those anisotropic flows in a real system, we simulate the Au + Au collisions at 200 GeV by AMPT with string melting model [16, 17]. A partonic phase is implemented in the model and the elliptic flow data from RHIC are well reproduced by the model [20]. We generate 1.6 millions minimum bias events. Their centrality dependencies are presented in Fig. 1. The red stars, the blue triangles, and the black solid circles are the anisotropic amplitudes of total transverse momentum $v_2(P_t)$, multiplicity $v_2(N)$, and mean transverse momentum $v_2(\langle\langle p_t\rangle\rangle)$, respectively.

Figure 1 shows that they have similar centrality dependence, small at peripheral and central collisions, and largest at mid-central collisions. But their anisotropies are different. The anisotropy of mean transverse momentum is the smallest, the anisotropy of multiplicity is in the middle, and the anisotropy of total transverse momentum is the largest, as it counts the anisotropy from both multiplicity and transverse momentum distributions, as we discussed in their definitions.

So the azimuthal distribution of mean transverse momentum can measure the anisotropy of radial expansion, in additional to the multiplicity distribution. We suggest the measurement of azimuthal distribution of mean transverse rapidity. Usually, the transverse rapidity of a final state hadron is considered as a good approximation of its transverse rapidity at kinetic freeze-out [21]. It is defined similarly to that of mean transverse momentum.

$$y_{\mathrm{T}} = \ln\left(\frac{m_{\mathrm{T}} + p_{\mathrm{T}}}{m_0}\right), \tag{7}$$

where m_0 is the particle mass in the rest frame, p_{T} is transverse momentum, and $m_{\mathrm{T}} = \sqrt{m_0^2 + p_{\mathrm{T}}^2}$ is the transverse mass.

The mean transverse rapidity in a given azimuthal angle bin is defined as the summation of all particles' rapidities divided by the total number of
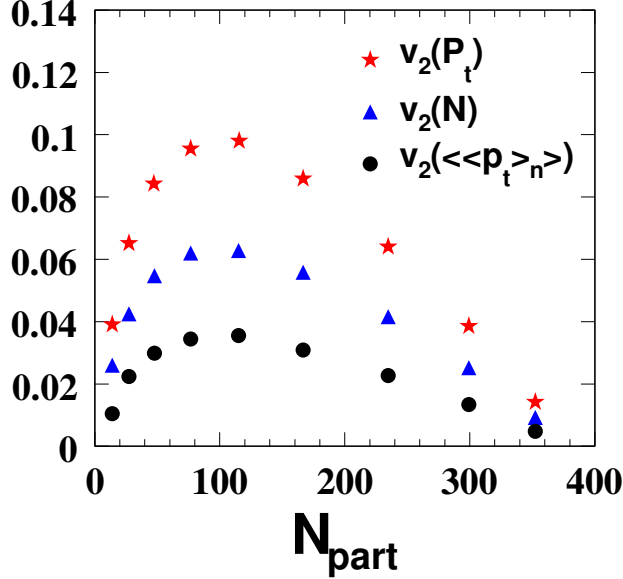
Fig. 1. (Color online) The centrality dependence of elliptic flow parameters deduced from azimuthal distributions of radial momentum (red stars), mean radial momentum (black solid circles), and multiplicity (blue solid triangles) for the sample of Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV generated by AMPT with string melting.

particles, i.e.,

$$\langle y_{\mathrm{T}}(\phi_m - \psi_r) \rangle = \frac{1}{N_{\mathrm{event}}} \sum_{e=1}^{N_{\mathrm{event}}} \frac{1}{N_m^e} \sum_{i=1}^{N_m^e} y_{\mathrm{T},i}^e(\phi_m - \psi_r), \qquad (8)$$

where the transverse rapidity of final state particle with mass m_0 is determined by its transverse momentum. Rapidity is more convenient in boost transformations. It should directly relate to the radial flow parameters that we are interested in.

The distribution of mean transverse rapidity in a minimum bias sample is shown in Fig. 2(a). We can see it is a periodic function of azimuthal angle, and consists of two parts. The isotropic constant, $y_{T0} = 1.3371 \pm 0.0001$, and azimuthal dependent part $y_{T2} = 0.0334 \pm 0.0002$.
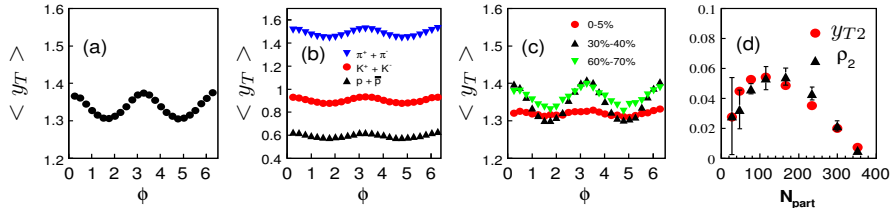
Fig. 2. (Color online) (a): $\langle y_T(\phi) \rangle$ for centrality $0-70\%$, (b): $\langle y_T(\phi) \rangle$ for 3 different mass particles, (c): $\langle y_T(\phi) \rangle$ at three different centralities, $0-5\%$ (red points), $30\%-40\%$ (black triangles), and $60\%-70\%$ (green down triangles), (d): The centrality dependence of y_{T2} (red points), and ρ_2 (black triangles).

3. Physics of the suggested measurement

The isotropic part of the suggested distribution contains both isotropic radial expansion and thermal motion. In order to see the features of the isotropic part, we study the mass dependence of the suggested distributions.

The suggested distributions of three different particles, pion (blue triangles), kaon (red points), and proton (black triangles) are given in Fig. 2(b). It shows that the distribution of the lightest pion is at the top with the largest isotropic rapidity, while the distribution of the heaviest proton is at bottom with the smallest isotropic rapidity, and the distribution of kaons with intermediate mass is between them with intermediate isotropic rapidity.

As we know, the thermal motion is mainly determined by the temperature and particle mass. At fixed temperature, the particles with small mass should have larger thermal velocity. So the isotropic part of the suggested distribution is just ordered as expected by thermal motion.

The features of the anisotropic part, the suggested distribution, at three typical centralities, 0-5% (red points For the most central collisions), 30%-40% (black up triangles for the middle-central collisions), and 60%-70% (green down triangles for the peripheral collisions), are given in Fig. 2(c).

It shows that the distribution is almost flat and azimuthal angle independent in central collisions, but azimuthal angle dependent in non-central collisions. It also shows the large anisotropy in mid-central collisions, and small anisotropy in peripheral collisions.

This centrality dependence is consistent with the fact that anisotropic radial flow appears in non-central collisions, and is the largest in mid-central collisions.

So the suggested distribution well represents the characters of radial

flow. Since we can not separate the thermal motion, the parameter of isotropic radial rapidity can not be obtained from the measurement. However, the thermal motion is isotropic, and has no contribution to the anisotropic part. The azimuthal dependent part, y_{T2}, should correspond to the parameter of anisotropic radial flow rapidity.

4. Measured anisotropic amplitude and extracted parameter of anisotropic radial flow rapidity

It is interesting to check if the measured anisotropic part corresponds to the extracted parameter of anisotropic radial flow rapidity.

The blast-wave model is currently the only model that simply includes the radial flow parameters. It is motivated from hydrodynamics with the kinetic freeze-out parameters [6, 12, 22, 24, 25, 26]. It is assumed that the longitudinal expansion is boost invariant [27]. The single-particle spectrum is given by the Cooper-Frye formalism (as in hydrodynamics) [28],

$$E\frac{d^3N}{d^3p} \propto \frac{1}{(2\pi)^3} \int_{\Sigma_f} p^\mu d\sigma_\mu(x) f(x, p), \tag{9}$$

where $f(x, p)$ is the momentum distribution at space-time point x. Eq. (4) is an integral over a freeze-out hyper-surface, and sums over the contributions from all space-time points.

Originally, local thermal equilibrium is assumed to be reached at kinetic freeze-out and a Boltzman distribution of the momentum is applied [12]. It has been shown recently that a Tsallis distribution provides an even better description for all p_T spectra from elementary to nuclear collisions [29, 30]. So, we use the Tsallis distribution for $f(x, p)$, i.e.,

$$f(x, p) = \left[1 + \frac{q-1}{T(x)}\left(p \cdot u(x) - \mu(x)\right)\right]^{-\frac{1}{q-1}}, \tag{10}$$

where q is the parameter characterizing the degree of non-equilibrium, and T is the kinetic freeze-out temperature. Thus the transverse momentum spectrum can be given by [31],

$$\frac{dN}{p_\mathrm{T} dp_\mathrm{T} d\phi} \propto \int_0^{2\pi} d\phi_s \int_{-y_b}^{y_b} dy e^{\sqrt{y_b^2 - y^2}} \cosh y \int_0^R m_\mathrm{T} r dr$$

$$\left[1 + \frac{q-1}{T}\left(m_\mathrm{T} \cosh y \cosh \rho\right.\right.$$

$$\left.\left. - p_\mathrm{T} \sinh \rho \cos(\phi_b - \phi)\right)\right]^{-\frac{1}{q-1}}, \qquad (11)$$

where m_T and p_T are transverse mass and transverse momentum of the particle, respectively, and $y_b = \ln(\sqrt{s_{NN}}/m_N)$ is the beam rapidity [32].

According to the generalized blast-wave parametrization, the radial flow rapidity which controls the magnitude of the transverse expansion velocity is [6, 22, 33, 34],

$$\rho = \widetilde{r}\big(\rho_0 + \rho_2 \cos(2\phi_b)\big) , \qquad (12)$$

where $\widetilde{r} = \sqrt{(r\cos(\phi_s)/R_X)^2 + (r\sin(\phi_s)/R_Y)^2}$. ρ_0 and ρ_2 are the parameters of isotropic radial flow rapidity and the amplitude of anisotropic radial flow rapidity, respectively. The greater the magnitude of ρ_2, the larger the momentum-space anisotropy. Here, ϕ_s is the azimuthal angle in coordinate space and ϕ_b is the azimuthal angle of the boost source element defined with respect to the reaction plane. They are related by $\tan(\phi_b) = (R_X/R_Y)^2 \tan(\phi_s)$.

There are 5 undetermined parameters: the temperature (T), isotropic radial flow rapidity (ρ_0) and anisotropic radial flow rapidity (ρ_2), q of the Tsallis distribution, and R_X/R_Y. Since all the particles are assumed to move with a common radial flow velocity, the mean kinetic freeze-out parameters are usually obtained by the simultaneous fitting of spectra from several hadrons [23, 24] and elliptic flow [6]. Elliptic flow, $v_2(p_\mathrm{T})$, is the second coefficient of the Fourier expansion of azimuthal multiplicity distribution [35, 18], and defined as,

$$v_2(p_\mathrm{T}) = \frac{\int_{-y_b}^{y_b} dy \int_0^{2\pi} d\phi \cos(2\phi) \frac{dN}{p_\mathrm{T} dp_\mathrm{T} dy d\phi}}{\int_{-y_b}^{y_b} dy \int_0^{2\pi} d\phi \frac{dN}{p_\mathrm{T} dp_\mathrm{T} dy d\phi}} . \qquad (13)$$

In Fig. 3, the p_T spectra of six particles, $\pi^\pm$, $K^\pm$, $\overline{p}$, and p, of the *same* sample, are presented by red solid circles. The differential elliptic flow $v_2(p_\mathrm{T})$ of pions, kaons, and protons are presented in Fig. 4 by black triangles, red solid circles and blue triangles, respectively. The error bars only include statistical errors. They are very small in comparison with the
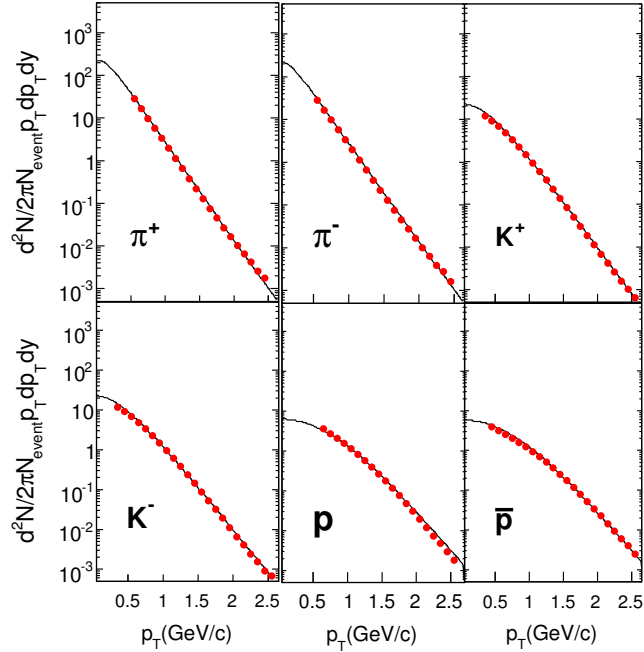
Fig. 3. (Color online) The transverse momentum spectra for $\pi^{\pm}$, $K^{\pm}$, $\bar{p}$ and p within $|y_{\mathrm{L}}| < 0.1$ at centrality $0 - 70\%$ for the sample of Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV generated by the AMPT model with string melting.

experimental data [22]. Typically, the systematic errors are considered to be 5% when fitting the simulated data [36]. Due to resonance decays in the low momentum region of pions [23], the data points in the low p_{T} regions of the spectra are excluded in this fitting.

Using Eqs. (6) and (8), the fitting curves in each plots of Fig. 3 and 4 are drawn. They describe well the corresponding data points of the p_{T} spectra and elliptic flow. The fitting parameters are $T = 102 \pm 1$ MeV, $\rho_0 = 0.67 \pm 0.01$, and $\rho_2 = 0.036 \pm 0.002$. This temperature is the same magnitude as that given by hydrodynamics [5, 10], and experimental data [22]. The parameter of anisotropic radial flow rapidity, ρ_2, is very close to the azimuthal amplitude of the suggested distribution, $y_{\mathrm{T}2} = 0.0343 \pm 0.0002$.

The centrality dependence of ρ_2 is shown in Fig. 2(d) by black triangles. We can see that at each centrality, ρ_2 is very close to $y_{\mathrm{T}2}$. The azimuthal amplitude of the suggested distribution coincides with the parameter of anisotropic radial flow rapidity extracted from a generalized
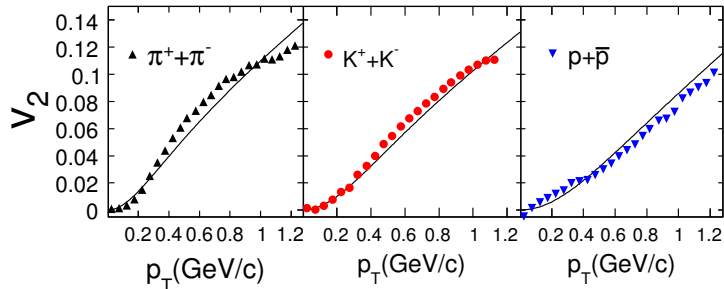
Fig. 4. (Color online) The differential elliptic flow $v_2(p_T)$ for different particle species within $|y_L| < 0.1$ at centrality $0 - 70\%$ for the sample of Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV generated by the AMPT model with string melting.

blast-wave parametrization. So they are consistent.

5. Summary

We suggest a measurement for the azimuthal distribution of mean transverse rapidity. It consists of two parts: isotropic, and anisotropic mean transverse rapidity. The isotropic part is the combination of isotropic radial expansion and thermal motion. It is demonstrated to be consistent with expected the mass ordering. The anisotropic part presents the anisotropic radial expansion. Its centrality dependence is shown to be consistent with extracted the parameter of anisotropic radial flow rapidity. The suggested distribution provides a model independent way to get the parameter of anisotropic radial flow rapidity. It is helpful for hydrodynamic calculations, and a model independent determination of shear viscosity [37].

Acknowledgments

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