Study of the helix model

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The quantum properties of a helix-like shaped QCD string are studied in the context of the semi-classical Lund fragmentation model. It is shown how simple quantization rules combined with the causality considerations in the string fragmentation process describe the mass hierarchy of light pseudoscalar mesons. The quantized helix string model predicts observable quantum effects related to the threshold behaviour of the intrinsic transverse momentum of hadrons, and of the minimal transverse momentum difference between adjacent hadrons.

1. Introduction

The concept of the QCD string with a helical structure has been introduced in [1] and some of its potential explored in [2]. The model has been shown to decribe the experimentally established correlations between the longitudinal and transverse momentum components of hadrons measured by DELPHI at LEP [3] and the azimuthal ordering of hadrons, recently observed by ATLAS at LHC ([4]).

The aim of this contribution is to discuss in some detail the space-time evolution of partons following a breakup of a QCD string with a helix structure. A concept of string quantization emerges from these considerations which has the merit to describe, in a consistent manner, several experimental observations.

2. Space-time properties of helical string model

In the transition from the 1-dimensional Lund string to a 3-dimensional helix-shaped string, it is necessary to reconsider some of the model properties. The basic assumption of a string modelling the confining QCD field with a constant string tension ($\kappa \sim 1$ GeV/fm) remains unchanged. However, the use of light-cone coordinates is no longer appropriate, as the trajectory of partons in the model is always bent by the interaction with the field.

In the case of slowly varying field, the string can be approximated by an ideal helix with radius R and constant pitch dΦ/dz, where Φ stands for the azimuthal angle (helix phase) and z is the space coordinate parallel to the string axis. Movement of a parton along the string can be thus described with the help of the longitudinal coordinate z and the folded transverse coordinate RΦ (Fig. 1).

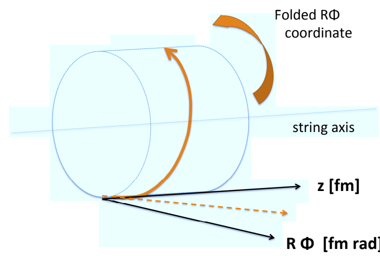

Fig. 1. The 2-dimensional coordinate system describing a helix-shaped string consists of the longitudinal string axis z and the folded transverse coordinate $R\Phi$, where R stands for radius of the helix and the Φ indicates the helix phase.

Following a string breakup at $[Re^{i\Phi_B}, z_B, t_B]$ into a pair of massless partons created at rest, the partons will move along the string and acquire the momentum

$$\begin{aligned} p_{||}(t) &= \pm\kappa\beta\ c\ (t - t_B) \\ p_T(t) &= \pm\kappa R(e^{i\omega c(t-t_B)} - e^{i\Phi_B}). \end{aligned} \tag{1}$$

The longitudinal velocity of partons β is related to the angular velocity ω

$$\beta = \sqrt{1 - (R\omega)^2}, \tag{2}$$

(the light-cone coordinates are recovered in the limit case R$\omega = 0$).

The momentum of a direct hadron created by adjacent string breakups at $[Re^{i\Phi_i}, z_i, t_i]$, $[Re^{i\Phi_j}, z_j, t_j]$ is

$$\begin{aligned} E_h &= \frac{\kappa}{\beta}|(z_i - z_j)| = \frac{\kappa}{\beta}|\Delta z|, \\ p_{h,||} &= \kappa\beta(t_i - t_j) = \kappa\beta\Delta t, \\ p_{h,T} &= \kappa R(e^{i\Phi_i} - e^{i\Phi_j}), \end{aligned} \tag{3}$$

and its mass is

$$m_h = \kappa\sqrt{(\Delta z/\beta)^2 - (\beta\Delta t)^2 - (2R\sin\Delta\Phi/2)^2}. \tag{4}$$

There is a fundamental difference (well illustrated by Eq. 4) between the helical string model and the standard Lund string model in what concerns the causality relation between breakup vertices. In the standard

Lund string model, the creation of a massive direct hadron requires a space-like distance between breakup vertices ($\beta = 1, m_h > 0 \Rightarrow |\Delta z| > |\Delta t|$). This implies some amount of magic has to be applied to make the hadrons emerge from the fragmentation with the correct mass. It turns out the problem of the on-shell birth is easily resolved in the helical string model, where a time-like distance between breakup vertices is possible.

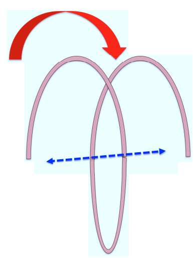

Fig. 2. The information about the string breakup propagates preferably along the string field, though a cross-talk between string loops is not excluded, either. The requirement of causal relation between breakups leads to an effective decoupling of the longitudinal and transverse component of the hadron momenta in the former case (see text).

In the following, the time-like distance between adjacent breakup vertices will be imposed - this is equivalent to the introduction of the causal relations in the Lund string fragmentation process. However, there is an ambiguity concerning the way the signal is allowed to propagate. If the information (about a breakup of the string at a given point) is allowed to pass along the string only, the space-time distance between adjacent vertices becomes negligible (to the extent we have neglected the parton masses) which means the propagating parton essentially triggers the following breakup and the mass of the outcoming hadron is (note that in this case $\Delta z = \beta \Delta t$)

$$m_S(\Delta\Phi) = \kappa R \sqrt{(\Delta\Phi)^2 - (2 \sin \Delta\Phi/2)^2}. \quad (5)$$

It is interesting to see that the longitudinal momentum is factorized out from the equation and that the hadron mass depends on the transverse properties of the string shape only. To obtain a discrete mass spectrum, it is sufficient to introduce quantization of the transverse coordinate RΦ (to be discussed in the following section).

There is of course also a possibility that the information about the breakup travels inside the string vortex (Fig. 2). To maintain the time-like difference between string breakups in such a case, the allowed time difference is then restricted to the interval

$$\sqrt{(\Delta z)^2 + (2R \sin \Delta\Phi/2)^2} \leq c\Delta t \leq \sqrt{(\Delta z)^2 + (R\Delta\Phi)^2} \quad (6)$$

and the outcoming hadron has a mass m_C in the range

$$m_S(\Delta\Phi) \leq m_C(\Delta\Phi) \leq m_S(\Delta\Phi)\sqrt{1+\beta^2} \quad (7)$$

(the subscripts S, C stand for "singular" and "continuous" mass solutions).

3. Mass spectra

Building on the causality requirements, we have obtained relations between the transverse string properties and the allowed hadron mass spectrum. It seems only natural to take a step further and to try to establish a quantization pattern for the string fragmentation which would match the measured discrete hadron mass spectra.

Let's assume the string quantization is realized through the quantization of the transverse coordinate

$$R\Phi \Rightarrow \mathrm{nR}\Delta\Phi = \mathrm{n}\xi, (\mathrm{n} = 1, 2, \ldots) \tag{8}$$

and that the n=1 case corresponds to the lightest hadron, the π meson.

Eq. 5 is particularly interesting for the study of light meson mass hierarchy because it describes the narrow pseudoscalar states (PS) decaying into an odd number of pions

$$\begin{aligned} PS &\rightarrow \mathrm{n}\pi,\ \mathrm{n} = (1), 3, 5, \ldots \\ m(PS) &= \kappa\sqrt{(\mathrm{n}\xi)^2 - (2\xi/\Delta\Phi)^2 \sin^2(\mathrm{n}\Delta\Phi/2)}. \end{aligned} \tag{9}$$

The results of the best fit matching the Eq. 9 to experimentally measured data [5] are listed in Table 1. Despite the fact that the simultaneous fit of 2 unknowns $(R, \Delta\Phi)$ from 3 hadronic states is overconstrained, a common solution describing the properties of the ground state is found. The π, $\eta(548)$ and $\eta'(958)$ masses are reproduced by Eq. 9 with precision better than 3% using $\xi = 0.192$ fm and $\Delta\Phi$=2.8 (for κ =1 GeV/fm).

$\kappa\xi$ [MeV]	κ R [MeV]	$\Delta\Phi$
192.5 $\pm$ 0.5	68 $\pm$ 2	2.82 $\pm$ 0.06
meson	PDG mass [MeV]	model estimate [MeV]
π	135 - 140	137
η	548	565
η'	958	958

Table 1. Best fit of the parameters of the pion ground state obtained from the mass spectrum of light pseudoscalar mesons. The η mass is reproduced within a 3% margin which serves as the base of uncertainty for $R, \Delta\Phi$ parameters.

Fig. 3 shows the dependence of the mass of PS mesons as a function of $\Delta\Phi$ in Eq.(9). With increasing $\Delta\Phi$, the predicted masses of η and η' reach the plateau (around $\Delta\Phi \sim 1.5$ rad) and lose sensitivity to the $\Delta\Phi$ value, but the mass of the π meson rises steadily till $\Delta\Phi \sim 5$ rad and effectively fixes the $\Delta\Phi$ value in the model.

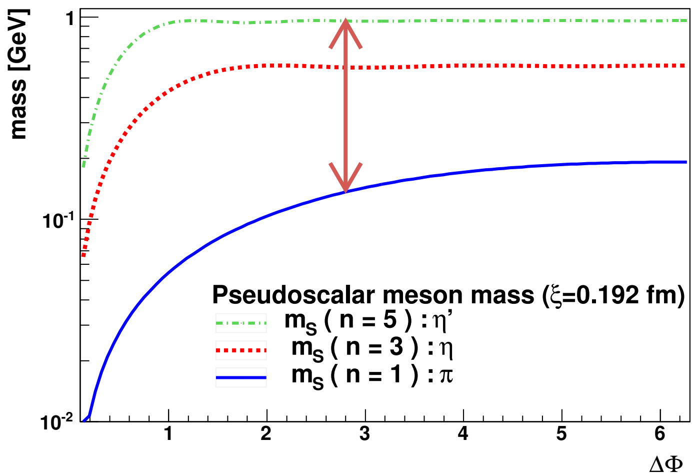

Fig. 3. The predicted masses of light pseudoscalar mesons as function of helix phase difference $\Delta\Phi$, for fixed $R\Delta\Phi$=0.192 fm rad.

The scalar nature of PS states is in agreement with the expectations of the quantization model: $m_T(PS)$ = n $m_T(\pi)$, thus the decay products of (η, η') have negligible (longitudinal) relative momentum in the rest frame of the mother resonance.

If the quantization model, in the first approximation, fits the mass spectra of light PS mesons, what can be said about the vector mesons (VM)?

The lightest vector mesons $\rho(770)$ and $\omega(782)$ can be interpreted as n=4 states decaying into m < n pions:

$$m_S(n = 4) = 0.76 \text{ GeV},$$

or n=3 states formed according to Eq. 7:

$$\sqrt{2}\ m_S(n = 3) \sim 0.79 \text{ GeV},$$

and their non-zero total angular momentum arises from the relative momentum of decay products (kinematically allowed since $m_T(VM) >$ m $m_T(\pi)$.

The mass of K*(890) and $\Phi(1020)$ mesons can be roughly associated with the mass of the $\rho(770)$ increased by the mass of the strange quark(s) (~120 MeV). (The same reasoning would classify K meson as a $n = 2$ state.)

It is worth noticing that the quantization of the transverse component of the string is equivalent to the quantization of the angular momentum J stored in the string (proportional to the transverse area spanned by the string) and that the relation

$$J \simeq \kappa(R\Delta\Phi)^2 = m_T^2/\kappa \tag{10}$$

indicates that the spectra derived from the model will lie along Regge trajectories.

4. Transverse momentum threshold

The discretization of the mass spectrum is not the only quantum effect which can be observed in the string fragmentation. In fact, the current investigation of the properties of the helix string quantization was prompted by the study of the inclusive p_T spectra. In [2] it has been shown that the helix string model significantly improves the description of the inclusive low p_T region. It has been also shown that the strength of the azimuthal correlations between hadrons can be described by the model only if the helix string model is extended to the decay of short-lived resonances. However, it turns out that such an extension spoils the agreement between the LEP data and the helix model essentially because the resonance decay according to the helix shaped "field memory" produces way too many low p_T particles (Fig. 4).

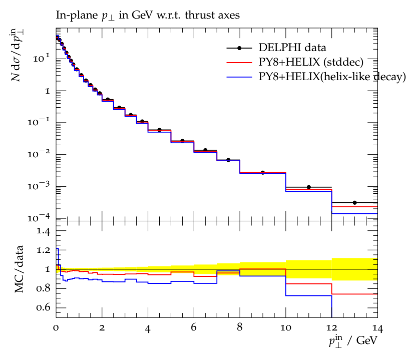

Fig. 4. Comparison of the helix string model predictions with the DELPHI data [3]. The production of low p_T charged particles is overestimated by the model when decay of short lived resonances is treated as a smooth continuation of the fragmentation of the helix string.

The effect cannot be tuned away as there are essentially no relevant free parameters left in the model. A careful study of the discrepancy and the gradients of the p_T spectra does not exclude existence of a natural p_T cutoff just below 0.2 GeV.

It is therefore encouraging to see - on the basis of results obtained in the previous section - that the production of soft pions with the $p \sim p_T <$ 0.14 GeV should be supressed in the quantized model. This result has yet to be propagated through the entire fragmentation and decay chain but this particular model feature is expected to help the regularization of the soft particle production in the helix string model extended to the decay of resonances.

5. Momentum difference of adjacent hadrons

The quantization of the helix string implies a quantization of the momentum difference between adjacent hadrons. Since the local charge conservation forbids the production of adjacent like-sign charged hadron pairs in the fragmentation process, the quantum effects can play a large role in the correlation phenomena with a significant difference between particle pairs with like-sign and unlike-sign charge combination.

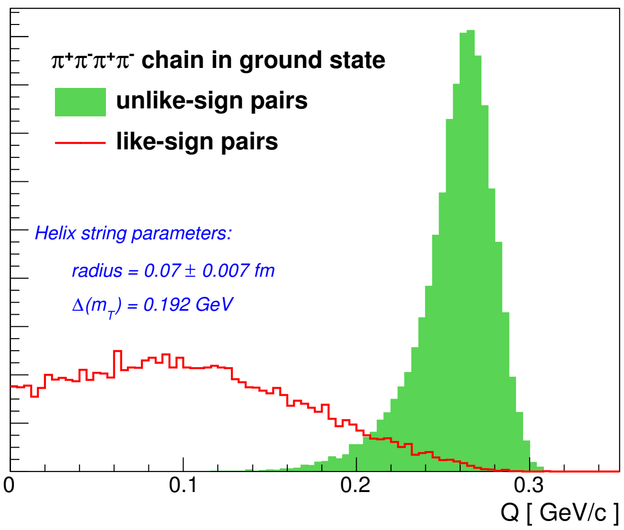

Fig. 5. The correlation pattern estimate for a chain of 4 charged pions in the ground state (Table 1). The longitudinal momentum differences are neglected, a variation of 10% is applied on the helix radius instead, in order to obtain a smooth spectra.

In the approximation of an ideal, or slowly varying helix string field, it is possible to make an estimate of the charge combination asymmetry induced by quantization. Consider a chain of adjacent charged pions in the ground state, for example from η' decay. The homogenity of the QCD field implies the difference between longitudinal momenta components of such pions is negligible. The momentum difference between pions along the chain is then given by the helix phase difference

$$\begin{aligned} Q &= \sqrt{-(p_i - p_j)^2} \approx 2p_T|\sin[0.5(\Phi_\mathrm{i} - \Phi_\mathrm{j})]| \\ &= 2p_T|\sin[0.5(\mathrm{j}-\mathrm{i})\Delta\Phi]|, \end{aligned} \tag{11}$$

where $p_T(\sim 0.14$ GeV) is the transverse momentum of the pion in the ground state, $i, j = 0, 1, 2, 3$ are integers corresponding to the rank of the hadron along the chain, and $\Delta\Phi(\sim 2.8)$ is the opening azimuthal angle between adjacent ground state pions. Fig. 5 shows the resulting correlation pattern with a marked separation of like-sign and unlike-sign pairs (the helix radius has been randomly varied by 10% in order to produce a smooth spectrum). The onset of excess of like-sign pairs occurs at $Q \sim 0.2$ GeV in the model. A large amount of experimental data provides evidence of an excess of like-sign hadron pair production in the low Q region. Most often, the data are studied from the perspective of the Handbury-Brown-Twiss model, i.e. as a signature of the incoherent particle production. The helix string model suggests an alternative point of view - such correlations may well be associated with fully coherent hadron production. In the specific case under study, due to the large opening angle $\Delta\Phi$, the quantized chain of ground state pions acquires properties reminiscent of Bose-Einstein condensate.

It should be possible to make a more precise experimental evaluation of the role of hadron 'chains' (and η' decay) in the correlation signal. Such a study may have a significant impact on the further development of the helix model, as it may confirm, or reject, the hypothesis of a strong link between resonance production and correlation phenomena.

6. Conclusions

The properties of the quantized helix string model have been investigated using a data driven simple quantization recipe. The model allows to introduce proper causal relations between the breakup vertices in the string fragmentation. The causality represents a strong constraint for the particle production and helps to understand the emergence of narrow hadronic resonant states. The fit of the light pseudoscalar mesons provides the parameters describing the ground hadronic state, and allows to make predictions concerning the threshold behaviour of relevant observables, to be verified with the help of experimental data.

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