DESIGN OF A 750-K. V. A. TURBO-ALTERNATOR

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ARMOUR INSTITUTE OF TECHNOLOGY
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turbo-alternator
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DESIGN OF A 750 kW, 3-φ 6600 v., 60 Hz, 4 pole TURBO-ALTERNATOR

In designing this machine we followed in main the system as outlined by Mr. Grey in his book on "Design of Electrical Machines."

We also consulted various books on electrical design, such as "Walker's "Design of Electrical Machines", Hobart and Ellis's "High Speed Electrical Machinery", Steinmetz's "Alternating Current Phenomena" and "Theory", and Calculations of electrical Circuits".

In designing a Turbo-Alternator, the size of rotor is limited by its speed, which according to standard practice, is 20000 ft. per minute at its maximum. The allowable largest diameter figures out to be 42.5 inches. After a few trial calculations, which are not given here for lack of space, it was decided on a diameter of 26".

The next condition to decide upon was the number of armature ampere-turns per inch periphery. This item is rather an experimental condition, as it is a result of various effects and the theor-
ical computation of it is too complex for practical purposes. However, it can be mentioned that the abnormal mechanical stresses set up in the coils under short-circuit conditions is one of the most important characteristics which limits the value of ampere-turns per inch periphery. Mr. Grey gives an experimental curve for armature ampere conductors per inch plotted against kilowatt output.

**DESIGN OF STATOR**

The number of ampere-conductors as obtained from the curve for 750 kw. rating is 475.

**STATOR AMPERE TURNS PER POLE**

\[
\frac{475}{2} \times p = \frac{475}{2} \times 20.4 = 4850
\]

where \( p = 20.4 \) is the pole pitch

Air-gap ampere-turns which according to Standard practice is:

\[
1.75 \times A_{T_s} = 1.75 \times 4850 = 8490 \text{ amp.turns.}
\]

**PROBABLE VALUE OF AIR-GAP**

\[
S = \frac{3.2 \times 8490}{B_{g \text{ max.}}}
\]
where $B_g\text{ max.}$ is the maximum gap density.

$B_g\text{ max.}$ is allowed to be 45000 for a turbo-generator.

It is obtained in the following manner:

$$B_g\text{ max.} = B_t\text{ max.} \times \frac{L_n}{L_c} \times \frac{t}{5}$$

The maximum permissible tooth density for turbos is found to be 100 lines per square inch. $\frac{L_n}{L_c}$ is approximately found to be 0.68; and $\frac{t}{L_c} = 1.5 \ldots$

$B_g\text{ max.} = 100000 \times 1.5 \times 0.68 = 42000$, then

$$S = \frac{3.2 \times 8490}{42000} = 0.65''$$

Diameter of Stator

$$26'' \times 2 \times 0.65 = 27.3''$$

Total conductors (approximate)

$$\frac{475 \times 3.14 \times 27.3}{67} = 608$$

67 is the full load current.

Stator pole Pitch

$$\frac{3.14 \times 27.3}{4} = 21.4$$

Slots per pole is assumed to be 18.
Total slots: \[ 4 \times 18 = 72 \]

Conductor per slot (actual): \[ 608 \div 72 = 8 \]

Total conductors (actual): \[ 8 \times 72 = 576 \]

Flux per pole:

To find the flux per pole we have to write out the e.m.f. equation

\[ E = 2.22 K \times a \times 10^{-8} \]  \hspace{1cm} (1)

"K" is the distribution constant depending upon the number of slots per pole and is .956.

\[ E = \frac{\text{terminal voltage}}{\sqrt{3}} = \frac{6600}{\sqrt{3}} = 3820. \quad \text{From equation (1)} \]

\[ a = \frac{3820 \times 10^8}{2.22 \times .956 \times 24 \times 60} = 5.3 \times 10^6 \]

SLOT PITCH

To find slot pitch divide number of slots into inside diameter of stator core.

\[ \text{slot pitch} = \frac{85.72}{72} = 1.19" \]
SLOT WIDTH

The slot width is taken approximately from an empirical curve which is derived from standard practice. The slot pitch in inches is the abscissae and slot width in inches the ordinate. Having previously found the slot pitch, then the slot width is taken from the curve which gives .7".

TOOTH WIDTH

The tooth width is the difference between slot pitch and slot width, which gives 1.19 - .7 = .49".

TOOTH AREA REQUIRED PER POLE

The minimum tooth area required is the average flux/pole divided by the average tooth density, or the average flux/pole divided by the maximum tooth density, which latter value is divided by 1.5, which is

\[ \frac{a_{\text{Bt max.}}}{1.5} = \frac{5.3 \times 10^6}{100000} = 90 \]

NET LENGTH OF IRON IN CORE

To find the net length of iron in core use the equation that the tooth area/pole = slots/pole \times \text{per cent enclosure} \times \text{minimum tooth width} \times \text{net}
axial length of iron.

\[ \text{In} = \frac{\text{tooth area/pole}}{\text{slots/pole} \times \text{percent enclosure} \times \text{min. tooth width}} \]

\[ \text{In} = \frac{90}{18 \times 0.7} = 7.13" \]

**GROSS LENGTH OF IRON IN CORE**

The gross length of iron in core is the net axial length divided by the constant 0.9.

\[ \text{Lg} = \frac{\text{In}}{0.9} = 8" \]

**CENTER VENT DUETS**

Use 3-one-half inch center vent-duets.

**FRAME LENGTH**

The frame length is equal to the gross length of iron in core plus the vent duets.

\[ \text{Lc} = 8 + 3 \times 1.5 = 9.5" \]

**AVERAGE GAP DENSITY**

The average gap density is equal to the flux per pole divided by the pole pitch times the frame length.

\[ \text{Bg Av.} = \frac{a}{\text{Lc}} = \frac{5.3 \times 10.6}{27.3 \times 9.5} = 20400 \text{ lines} \]
DESIGN OF ROTOR

Probable weight of rotor:

\[
\frac{d^2 \times Lc \times Sp.}{4} \times 1.5
\]

where \( Lc \) = length of rotor
\( d \) = diameter of rotor
\( Sp \) = specific gravity of iron = .28

\[
\therefore W = \left( \frac{26^2}{4} \times 9.5 \times .28 \right) 1.5 = 2200 \text{ lbs.}
\]

DIAMETER OF ROTOR SHAFT

The diameter of the shaft was determined by designing it of such size that its frequency of vibration is far below or above the natural frequency due to bending, or critical speed.

The expression for critical speed as derived by us, checks within a few per cent with that as given by Mr. Behrend, namely,

\[
r \cdot p \cdot m \cdot = 72 \sqrt{\frac{EI}{M \cdot L^3}}
\]

where \( E \) = modulus of elasticity
\( I \) = moment of inertia at section about diameter of shaft
\( M \) = mass of rotor
\( L = \text{distance between bearing} \frac{2}{2} \)

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By algebraic transformation we get:
\[ r \cdot p \cdot m. = 100 \, d^2 \sqrt{\frac{28 \times 10^6}{\text{rotor} \, \sqrt{\omega \cdot t \cdot x \, L^3}}} \]

Substituting for critical speed a value twice the actual frequency, that is, let \( r \cdot p \cdot m. = 3600 \), we obtain,
\[ d_s = 7.5'' \]

The deflection must also be limited to a value of 5% of air-gap so that there should be no trouble due to magnetic unbalancing.

\[ \text{Defl.} = \frac{W \times (2 \, L)^3}{48 \, E \, I} = 0.05 \, S \]

Substituting
\[ W = 2200 \]
\[ L = 22.5 \]
\[ E = 3 \times 10^7 \]
\[ I = \frac{d^4}{64} \]

We get:
\[ \text{Deflection} = \frac{2200 \times 45^3}{48 \times 3 \times 10^7 \times 7.5^4} = 0.0065'' \]

which is only 1% of the air-gap clearance.

The stress set up in the shaft due to bending
and twisting moments is found from the formula:

\[ Me = \text{equia. bend. mom.} = \frac{M_b + \sqrt{M_b^2 + M_t^2}}{2} \]

where \( M_b = \text{bend. mom. at center of shaft} = \frac{W}{2} \text{ in. lb.} \)

\[ M_t = \text{twisting moment} = \frac{\text{watts input} \times 85 \text{ inch. lbs.}}{\text{r.p.m.}} \]

\[ \cdot \cdot \cdot \quad Me = M_b + \frac{M_b^2 + M_t^2}{2} = S \times \frac{x}{32} \times d_s \]

\[ M_b = \frac{2200 \times 22.5}{2} = 24200 \]

\[ M_t = \frac{830 \times 85}{1800} = 41 \text{ in. lb.} \]

\[ \frac{d^3}{32} = \frac{x \times 7.5^3}{32} = 41.5 \]

\[ \cdot \cdot \cdot \quad Me = 24200 + \sqrt{24200^2 + 41.5^2} = 41.5 \times S \]

\[ \cdot \cdot \cdot \quad S = 600 \text{ lbs. per sq. inch.} \]

which is an exceedingly soft value.

**DEPTH BELOW SLOTS (d₂)**

\( d_2 \) is obtained from the formula:

\[ d_2 = \frac{(r_1^3 - r_3^3) \times \text{r.p.m.}^2}{3.8 \times 14000 \times 10^5} \]
where

\[ r_1 \text{ = external radius of rotor} \]
\[ r_3 \text{ = radius of shaft} \]

\[ d_2 = \frac{(13^3 - 3.75^3) \times 1800^2}{3.8 \times 14000 \times 10^5} = 1.06'' \text{ (approx.)} \]

This value is not final but must be checked for the stresses set up in teeth, namely:

Assuming 6 slots per phase per pole and a slot width of .75", max. unit stress as 6000, we solve for tooth width from following formula:

\[ 6000 = \frac{(r_1^3 - r_2) \times \text{r.p.m.} \times (t + S) \times \frac{360}{21.5 \times 10^6 \times t}}{r_2} \]

where \( r_2 \) = distance between bottom of slot and center.

We obtain for \( t \)

\[ t = .5'' \]

This value can only be realized by making

\[ d_2 = 3.23''. \]

**ROTOR SLOT DEPTH (\( d_s \))**

\[ l_s = 13 - 3.75 - 3.25 = 6'' \]

Probable depth of wedge is .25''.

Available slot depth 6 - .25 = 5.75''.
Stator amp.-turns per pole

\[ \frac{572 \times 67}{8} = 4810 \]

Maxim. field amp.-turns at full load \(3.25 \times 4810\) = 15700 amp.-turns.

The constant 3.25 is given by Grey as a good value from modern practice.

**LENGTH** OF MEAN TURN AS OBTAINED FROM SCALE-DRAWING

\[ MT = 60" \]

Section of rotor conductor

\[ Sc = \frac{15700 \times 60}{27.5} = 33000 \]

adding 10% to it for conduction

\[ 33000 + 0.1 \times 33000 = 37000 \]

The maximum current in the rotor is determined by heating conductions. Grey gives the following formula:

\[ \text{circ. mill/amp.} = \frac{\text{amp. cond./pole} \times \frac{10}{5} + \frac{33}{2d}}{\text{pole-pitch} \times \text{slots/pole}} \]

where \(d\) = depth of slot.

From this formula we obtain:
circ. mills per amp. = 354

Hence, max. current I

\[ I_{\text{max.}} = \frac{37 \text{ Total circ. mills}}{\text{core mills/amp.}} \]

\[ = \frac{37000}{354} = 91 \text{ amp.} \]

Ampere-Conductors per slot:

\[ \frac{15700 \times 2}{6} = 5233 \]

Conductors per slot:

\[ \frac{5233}{91} = 56 \]

Assuming 7 conductors per coil per slot, we get

\[ \frac{56}{7} = 8 \text{ coils.} \]

**STATOR CORE DESIGN**

Conductors per slot was found to be equal to 8.

Amp. cond./slot

\[ 8 \times 67 = 536 \text{ amp. cond.} \]

where 67 is full load current.

Amp. Conductors per inch.

\[ \frac{536 \times 72}{3.14 \times 27.3} = 453 \]
Circular mills per ampere assumed for first approximation is 800.

Circular mills per conductor 800 x 67 x 53600.

Section of conductor:

\[
\frac{53600 \times 3.14}{10^6 \times 4} = 0.042''
\]

The nearest size in a d.c.c. wire is one with diameter = .24''.

Assuming 4 layers per slot we have 4 x .24 = .96'' for radial depth of occupied with copper.

The depth of wedge is .25''.

Insulation between layers 3 x .02 = .06.

Hence, total depth of slot is 1.59 or 1 9/16''.

The thickness of insulation between vertical walls of slot and copper is found as follows:

Width of slot .70''.

Width occupied with copper 2 x .24 = .48''.

Thickness of insulation.

**TEMPERATURE DIFFERENCE BETWEEN COPPER AND IRON**

\[
\frac{453}{800} \times \frac{.22}{2 \times 1.59 \times .7} \times \frac{1}{.003} = 10^0 C.
\]
This formula has been taken from Grey's treatise on electrical machine design.

DEPTH OF IRON BEHIND SLOTS

This value is obtained in the following manner:

Depth of iron behind slots =

\[
\frac{\text{flux per pole}}{2 \times \text{flux density} \times \text{net length}} = \frac{5.3 \times 10^6}{2 \times 65000 \times 7.13} = 5.7\text{" or } 5\frac{22}{32}\text{".}
\]

CALCULATION OF FIELD CURRENT AT FULL-LOAD Cos. = .8.

The flux per pole is \(5.3 \times 10^6\) when \(E_g = 6600\).

The gap density is then:

\[
B_g = \frac{5.3 \times 10^6}{21.4 \times 9.5} = 26000
\]

The gap amp.-turns

\[
A_{tg} = \frac{2600}{3.2} = 5300
\]

The demagnetizing ampere-turns of armature reaction at .8 power factor is \(= .6 \times \) (amp.-turns at 0 power factor).

The formula for armature-reaction is \(A_T = .275 \times \text{cond. per pole} \times I\).
\[ 0.275 \times 144 \times 67 = 2630 \]

Hence, the number of amp.-turns required to produce the full-load flux will be:

\[ AT = 5300 + 2630 \times 0.6 = 6900 \]

where \( 0.6 = \sqrt{1 - \cos^2} \)

This is the average number of amp-turns required. But, in fact the ampere-turns on a cylindrical rotor is a variable quantity and is maximum at the center of the pole and zero at the middle of the pole pitch. It has a sine wave form. It can be seen from the assembly drawing that it is the maximum value of the sine wave which determines the average number of ampere-turns.

To get the maximum value we divide the average by \( \frac{2}{\pi} \)

\[ AT_{\text{max.}} = \frac{6900}{0.636} = 10900 \]

There are 168 turns per pole.

Hence, \( I_c = \frac{10900}{168} = 65 \text{ amps.} \)
CALCULATION OF EFFICIENCY

Resistance of armature per phase at 60° C. is .16.

Copper loss in armature

\[ \text{C.L.} = \frac{3 \times .16 \times 67^2}{10^3} = 2.13 \text{ kw.} \]

Field loss

\[ \text{F.L.} = \frac{110 \times 65}{1000} = 7.15 \text{ kw.} \]

IRON LOSS

This loss is calculated from an empirical curve, which the loss in watts per pound of iron.

Weight of core:

\[ W = .26 \times 36 \times 5.7 \times 9.5 \times 3.14 = 15400 \text{ lbs.} \]

The loss per pound at 60 cycles is 6.5.

Hence, the iron loss is:

\[ \text{P}_i = \frac{15400 \times 6.5}{1000} = 10 \text{ kw.} \]

The windage and friction loss is found to be 6.4 kw.

Hence, the total loss:

\[ \text{P}_i = \text{P}_a + \text{P}_f + \text{P}_i + \text{P}_w = 2.13 + 7.15 + 10 + 6.4 = 25.68 \text{ kw.} \]
The efficiency is equal:

\[
\text{Eff.} = \frac{\text{output}}{\text{input}} = \frac{600}{600 + 25.68} = 95.6\%
\]

**CALCULATION OF REACTANCE DROP PER PHASE**

Let \( C \) = number of cond. per phase per pole

\( p \) = number of poles

\( P_i \) = permeance in iron/cm.

\( P_a \) = permeance in air/cm.

\( P_e \) = permeance in end connections/cm.

\( l_i \) = net length of core

\( l_a = 3 \times d \), where \( d \) = diameter of vent duct.

\( l_e \) = length of one end connection.

\( L \) = inductance/phase/pole.

Then

\[
L = p \cdot C^2 (P_i \cdot l_i + P_a \cdot l_a + P_e \cdot l_e) \times 10^{-8}
\]

\[
= 4 \times 48^2 \times 2.54 \times (4.5 \times 7.13 + .5 \times 1.5 + 5 \times 45) \times 10^{-8}
\]

\[
= .013 \text{ henrys.}
\]

The \% reactance drop is then

\[
100 \times \frac{2 \times 60 \times .013 \times 67}{6600} = 4.96%.
\]

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### SPECIFICATIONS FOR 750 kva., 6600 v, 3- , Cos. = 0.8.

**TURBO-ALTERNATOR**

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output in kw.</td>
<td>600</td>
</tr>
<tr>
<td>Output in kva.</td>
<td>750</td>
</tr>
<tr>
<td>Power factor</td>
<td>0.8</td>
</tr>
<tr>
<td>Terminal voltage</td>
<td>6600</td>
</tr>
<tr>
<td>Style of connection</td>
<td>Y</td>
</tr>
<tr>
<td>Current per terminal</td>
<td>67</td>
</tr>
<tr>
<td>Speed in r.p.m.</td>
<td>1800</td>
</tr>
<tr>
<td>Frequency</td>
<td>60</td>
</tr>
<tr>
<td>Number of poles</td>
<td>4</td>
</tr>
</tbody>
</table>

**ARMATURE IRON**

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter at air-gap</td>
<td>27.5</td>
</tr>
<tr>
<td>Diameter at bottom of slots</td>
<td>31.5</td>
</tr>
<tr>
<td>External diameter of laminations</td>
<td>42.3/8</td>
</tr>
<tr>
<td>Number of vent. ducts</td>
<td>7</td>
</tr>
<tr>
<td>Width of each duct</td>
<td>0.5</td>
</tr>
<tr>
<td>Effective core length</td>
<td>7-1/8</td>
</tr>
<tr>
<td>Width of vent. ducts</td>
<td>3.5</td>
</tr>
</tbody>
</table>
### SLOTS AND TEETH

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of slots</td>
<td>72</td>
</tr>
<tr>
<td>Slot pitch at arm. core</td>
<td>1-3/16</td>
</tr>
<tr>
<td>Width of slot</td>
<td>11/16</td>
</tr>
<tr>
<td>Width of tooth at arm. core</td>
<td>8/16</td>
</tr>
<tr>
<td>Radial depth of slot</td>
<td>1-9/16</td>
</tr>
</tbody>
</table>

### ARMATURE COPPER

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of slots per pole/phase</td>
<td>6</td>
</tr>
<tr>
<td>Number of conductors per slot</td>
<td>8</td>
</tr>
<tr>
<td>Section of conductor</td>
<td>.042&quot;</td>
</tr>
<tr>
<td>Current in amperes</td>
<td>67</td>
</tr>
<tr>
<td>Current density in amperes/sq.cm.</td>
<td>244</td>
</tr>
<tr>
<td>Diameter of core conductor</td>
<td>.32&quot;</td>
</tr>
<tr>
<td>Number of turns in series/phase</td>
<td>96</td>
</tr>
</tbody>
</table>

### ROTOR IRON

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter at pole core</td>
<td>26&quot;</td>
</tr>
<tr>
<td>Length of air-gap</td>
<td>21/32</td>
</tr>
<tr>
<td>Pole pitch at air-gap</td>
<td>20.4</td>
</tr>
<tr>
<td>Effective axial length of rotor</td>
<td>7-1/8</td>
</tr>
<tr>
<td>Total number of slots</td>
<td>56</td>
</tr>
<tr>
<td>Number of turns per pole</td>
<td>168</td>
</tr>
</tbody>
</table>
Section of conductors ....037" 
Current in amperes 65
Current density 250
Diameter of conductor ....216
Total area of copper 2.08 "
Space factor ....46
Total area of slot 4.5 "
Mean length of turn 60
Resistance of all coils at 60° C. ....442
Volts across field 110

Flux per pole 5.3 x 10^6

MAGNETIC DENSITIES (sq. inch)
Armature core 65 x 10^3
Teeth (maximum) 10^3

LOSSES
Armature Copper
Mean length of turn 90
Resistance per phase at 60° C. ....16
Copper loss 2.13 kw.
ARMATURE IRON

- Wt. of armature Iron (no teeth) | 15400
- Frequency | 60
- Watt-loss per pound | 6.5
- Iron loss | 10 kw.

FIELD COPPER

- Field current | 65
- Field loss | 7.15
- Windage & friction loss | 6.4
- Total Loss | 25.68

- Efficiency at full-load, Cos. = .8 | 95.6
- Per cent reactance drop | 4.96