DESIGN AND TEST OF AN ELECTRIC FURNACE PYROMETER

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DESIGN AND TEST OF AN ELECTRIC FURNACE PYROMETER

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According to latest estimates (1920), almost a billion dollars worth of coal is wasted every year in the United States due to inefficient combustion of fuels under steam boilers. The average boiler efficiency is around 50% and could be easily raised to around 70% if the biggest loss due to imperfect combustion could be eliminated.

The problem then resolves itself into finding a method for determining the proper proportioning of air and fuel in the furnace. This method must be simple enough for the fireman to follow, and the more independent the device used, as a guide, is of local conditions the more its practicability.

Now, if the temperature of the furnace, or rather of the gases leaving the furnace, be studied, it will be found that the closer the amount of air used for combustion is to theoretical requirements the higher will the furnace temperature be. In fact, as will be shown later, for all fuels satisfying the condition that they require
7.5 lbs. of air or thereabouts for each 10,000 Btu. of their heat value, there is a definite maximum temperature for the gases in the furnace for perfect combustion. If 200% air is used, the temperature of the furnace will be about half its maximum, and, in general, the temperature will be practically inversely proportional to the relative amount of air used.

The furnace temperature is, therefore, a guide to combustion efficiency. Thus suppose that it is desired to avoid clinker formation and the burning out of arches, so that a furnace temperature of 2000°F is decided upon as the ideal one. Then, if for the fuel used, the maximum theoretical temperature is 4000 °F, the chosen temperature corresponds to about 200% air or to a 100% excess. If 7.5 lbs. of air is the theoretical requirement for 10,000 Btu., and if the boiler room and flue gas temperatures are, say, 80°F and 480°F, respectively, the loss due to excess air, for an average specific heat of .25, is,

\[
\text{Excess Air losses} = 0.25 (7.5)(480 - 80) = 850 \text{ Btu.}
\]
For 200% air, the loss is doubled and is, therefore, increased to 17%. Considering the fact that the usual excess air found in practice is around 300–400%, the losses due to excess air are reduced from 32 or 42.5% to only 17%, with a resulting saving of about 20%, if the furnace temperature is maintained at 2000°F by damper regulation with a furnace pyrometer as a guide.

A simple reliable furnace pyrometer would, therefore, make possible tremendous fuel savings, if followed by the fireman. The difficulties of using thermo-couple pyrometers for measuring furnace temperatures are twofold. In the first place the temperatures of the gases in the furnace vary by several hundred degrees from point to point, so that the indications of one thermo-couple in a furnace of say 100 square feet of grate, surface would be meaningless on account of such poor "sampling efficiency".

In the second place, the average thermo-couples are not suited for continuous exposures to temperatures ranging from 2000 to 3500°F.
Optical pyrometers are not satisfactory chiefly because they indicate the temperature of the fuel bed instead of the gas temperature. In fact a variation of about 800°F in the gas temperature has been found to correspond to only a small change of about 200°F in the fuel bed temperature.

For this reason a new electric furnace pyrometer has been devised of a high "sampling efficiency" and of great sensitivity and simplicity. This thesis comprises a discussion of the theory design and test of this type of furnace pyrometer which was installed on a 350 H. P. Stirling Boiler at the Armour Institute steam power plant.
GENERAL DESCRIPTION AND THEORY

In practically every furnace, the hot gases pass over the bridge wall before reaching the boiler tubes or the flues, as the case may be. The average temperature of the gases as they pass over the bridge wall is therefore, the one that would indicate the combustion efficiency of the furnace.

Suppose now that a conducting, say metal, tube is passed through the setting above the full length of the bridge wall and water is allowed to flow continuously through this tube at a constant rate. Then the rise in the temperature of the water due to the effect of the hot gases on the tube is evidently an indication of the average temperature of the hot gases along the full length of the bridge wall.

The test pyrometer used on the Sterling Boiler consisted of 3/4" wrought iron pipe stretched across the bridge wall and supported by a 24" wall on one side and by about a 36" wall on the other side. It was connected to a city water main, and discharged into the sewer, the flow being regulated by means of a valve.

The design of such a pyrometer involves two dis-
distinct phases.

(1) The thickness, diameter, length and material of the tube must be such that the final water temperature for the highest furnace temperature to be measured shall be safely below boiling point. The rise of the water temperature should be between 25 and 100°F to be conveniently measured, and the heat absorbed by the pyrometer tube should be as small as possible, or only a fraction of a percent of the boiler rating. The coefficient of heat transmission should bear a definite constant relation to the temperatures measured.

(2) The thermal e.m.f., if thermocouples are used, should be sufficient to make it possible to use a comparatively rugged millivoltmeter. If a recording pyrometer is desired, means must be found to measure something which can be recorded, e.g., resistance.

The problem then is both a mechanical and electrical one.

Analyzing the problem of heat transmission first, we have the relation:

(1) \( \text{US} (\text{tg} - \text{tw}) = \text{Heat absorbed by tube,} \)
(2) \[ 60 \text{WC} \left( t_2 - t_1 \right) = \text{Heat given up to water,} \]
where

\[ U = \text{Coeff. of heat transmission } \frac{\text{Btu.}}{\text{Hr.} \times \text{Sq. Ft.} \times ^\circ F.} \]

\[ S = \text{Tube Surface, Sq. Ft.} \]

\[ t_g = \text{Temperature of gases in furnace} \]

\[ t_w = \text{Mean temper of water in tubes} \]

\[ t_2 = \text{Temper of water, hot end} \]

\[ t_1 = \text{, cold end} \]

\[ w = \text{Water dischage, LBS./MIN} \]

\[ C = \text{Specific heat of water} = 1 \]

Since there are practically no losses, we have,

(3) \[ US \left( t_g - t_w \right) = 60 \ W \left( t_2 - t_1 \right) \]

For an average value of \( U = 20 \), as commonly used for boiler tubes, and for the pyrometer tube furnace temperature etc. as used on the Stirling boiler, we have, \( U = 20 \),

\[ S = \pi d l; \ d = 1^" \text{ (O.D.)}; \ L = 9'3^\circ = 9.25' \]

or

\[ S = \frac{9.25 \pi}{12} = 2.42 \text{ sq. ft.} \]

\[ t_g = 2000^\circ F. \]

\[ t_w = 100^\circ F. \]

\[ w = 60 \text{ av D; a} = .533 \text{ sq. in.} \]

\[ V = 1.5 \text{ ft/sec.} \ D = 62 \# / \text{cu.ft.} \]
or

\[ w = \frac{60 \times 0.533 \times 1.5 \times 62}{144} = 20.7 \text{ lbs/min} \]

Therefore, the rise in water temperature is,

\[ T_2 - t_1 = \frac{\text{US} \times (t_g - t_w)}{60^W} = \frac{20 \times 2.42 \times 1900}{60 \times 20.7} = 74^\circ F. \]

The water is, therefore, safely below boiling point for the case considered. If the furnace temperature is much higher than 2000°F, the water rise may be still kept down by increasing the flow. Thus, if the velocity is increased from 1.5 to say 3 ft./sec. the temperature of the furnace could be in the neighborhood of 3900°F for the same water temperature rise of 74°F.

It is also evident that the above water temperature rise is large enough for electrical measurement. However, if we analyze the assumed design for heat absorbed, we find that,

Heat absorbed = \text{US} \times (t_g - t_w) = 20 \times 2.42 \times 1900 =

\[ 92,000 \text{ Btu/Hr} = 92,000 = \frac{33,500}{2.75} \text{ Boiler Horse Power.} \]

or

\[ \frac{2.75}{100} \times 350 = 0.785\% \text{ Boiler rating which is unnecessarily too large.} \]

By proper choice of the material and the dimen-
sions of the pyrometer tube, the heat absorbed may be considerably reduced without appreciably affecting the water temperature rise or the sensitivity of the pyrometer. For, rewriting (3) we get,

\[ U dl (tg-tw) = 3600 \alpha v D (t_2-t_1) = \]

\[ 3600 \frac{\pi (kd)^2}{4x144} 62v (t_2t_1) \]

or

\[ VL (tg-tw) = 387k^2 d v (t_2-t_1). \]

where \( K \) is the ratio of inside diameter to outside diameter

(4) From the last equation we note that if \( U \) is decreased we must only decrease either \( K^2 d \) or \( v \) in proportion, to still retain the same water temperature rise, \( t_2-t_1 \).

Now sensitivity may be considered as the time it takes for a change in furnace temperature to be recorded by the pyrometer. For a given lag of heat transmission through the pyrometer tube, the time considered, must evidently depend upon the time it takes for the water to pass through the full length of the tube. Therefore, the greater the velocity of flow, the shorter this time is and the greater the sensitivity. Therefore, to reduce the heat absorbed without affecting either sensi-
tivity or rise of water temperature, the proper method is
to decrease the coefficient of heat transmission $U$, and
the diameter, $d$, of the pyrometer tube or the expression,
$k^2d$ in equal proportion.

It may be remarked, in passing, that for the py-
rometer tube under discussion, the time it takes for a
particle of water to move from the cold to the hot end is,

$$ t = \frac{1}{V} = \frac{9.25}{1.5} = 6.16 \text{ sec.} $$

If a change of furnace temperature occurs at the
hot end of the pyrometer tube the time it takes for the
effect to be recorded is zero. The mean time for the rec-
ording of furnace temperature changes at any point of the
tube must therefore, be,

$$ t_2 = \frac{6.16 + 0}{2} = 3.08 \text{ sec.} $$

Hence the gratifying sensitivity of this type of
furnace pyrometer.

Before redesigning the tube, let us rewrite equa-
tion 1 and 4 in terms of internal diameter $d_i$, of the tube,
and thickness, $T=nd_1$. Thus:

\begin{align}
(5) \quad d &= d_i + T = d_i(1+n), \\
(1a) \quad U = \frac{c(t_c - t_w)}{V} \text{dI} \quad (t_c - t_I) = \frac{U}{K} \text{dI} (1+n)L \quad (t_c - t_w) = \text{Heat absorbed}
\end{align}
\( (4_a) \, U_{L(t_2-t_w)} = 387 \, \frac{d_1^2}{d} \, \sqrt[3]{(t_2-t_1)} = 387 \, \frac{d_1}{1+n} \, \sqrt[3]{(t_2-t_1)} \)

Suppose, now, that the inside diameter, \( d_1 \), is reduced from 3/4" to \( \frac{5}{8} \)", and \( U \) is decreased in proportion or to 1/3 its original value. Then there will be no change in either sensitivity or water temperature rise, but there will be a large reduction in heat absorbed, since in eq. \((1_a)\) reduced to 1/3 their former value. The reduction is therefore to 1/9 of the former heat absorbed, and we have,

\[ \text{Heat absorbed} = \frac{1}{9} (92,000) = 10,200 \, \text{Btu/hr} = \frac{10,200}{33,500} = 0.3 \, \text{Boiler Horse Power} \]

The value of \( U \) can be reduced by increasing the thickness of the pyrometer tubes and by choosing a material which is a cross between a good heat conductor and an insulator. It is thought that alundum, the earthen material, which has lately come into use for protecting pyrometer thermocouples, is such a material, inasmuch as it certainly has a much smaller heat conductivity than metal, but still has not so much heat transmission as to be unsuited for pyrometry.

The experimental investigation of the furnace pyrometer under discussion was planned along the following lines:
(1) The relation between furnace temperature distribution and the furnace pyrometer was to be investigated for different temperature ranges. The coefficient of heat transmission, $U$, was to be found for different furnace temperatures, with the velocity of the water in the pyrometer tubes being kept constant. The effect, if any, of soot or fused ash accumulations of the pyrometer tube on the coefficient of heat transmission, $U$, was to be found; also, the variation, if any, of $U$, with the velocity of the water in the pyrometer tube.

(2) The relation between heat transmission, from a hot gas to water through a tube, and the thickness of the tube was to be sought. Seamless steel and alundum tubes were to be experimented with.

Because of the great time consuming nature of investigating furnace temperature distribution, all readings were taken only for a constant water velocity of about, 1.5 ft/sec. Preparations were made for part (2) of the test, but the electric furnace used could not produce a temperature of 2000°F, as was planned. It was calculated that it would be necessary to almost double the platinum ribbon winding of the furnace in order to consume the full 3KW. of the furnace rating, since the current was limited to 30 amperes. Various factors made this plan unattainable within the period
available for the investigation, and the latter had to be postponed for a future time.

However, the method of analyzing the effect of tube thickness on the coefficient of heat transmission is given below, the derivations of the basic relations being given in the appendix.

Before taking this matter up, it is interesting to note what the effect, according to eq. (4a) and (1a), would be if the thickness of tube were varied for a given internal diameter \( d_i \). Thus, if the thickness were increased, and therefore also \( l+n \), then according to eq. (1a), \( U \) would have to be proportionately decreased in order that the heat absorbed should not increase also. Increasing of thickness would decrease \( U \) but in all probably much less than in proportion to \( l+n \).

Therefore, for a given material with a fixed internal diameter, increase of thickness would probably also increase the heat absorbed by the tube. If the changes of \( l+n \) and \( U \) were universally proportional to each other, then according to eq. (4a) the water temperature rise would not be affected thereby. But neither would the heat absorbed be decreased. Therefore, at the best there would be no gain. In fact, since, in all probability, the coefficient, \( U \) would be decreased but slightly in comparison with the effect of increase of thickness upon the increase of \( l+n \), the net result would be an in-
crease in water temperature rise, \( t_2 - t_1 \). Although the latter is in itself desirable, it is not so at the expense of increase in heat absorbed. However, just what relation does increase in thickness bear to decrease in \( V \) is something to be found yet.

It is shown in the appendix that if, \( t_g \) = mean temperature of the gas, \( t_{mo} \) = mean temperature of tube, outside

\[
\begin{align*}
t_{mi} &= \quad \text{inside} \\
t_m &= \quad \text{at any thickness, } T, \text{ from inside surface.}
\end{align*}
\]

\( t_1 \) = temperature of water, cold end

\( t_2 \) = temperature of water, cold end

\( t \) = temperature of water, cold end.

\( d \) = diameter of any point inside the metal

\( U_1 \) = coefficient of heat transmission, from gas to metal

\( U_2 \) = coefficient of heat transmission, from metal to liquid

\( C \) = metal conductivity.

that the fact that as much heat is transmitted from the gas to the metal as through the metal as well as from the metal to the water can be expressed by the following three equations:

\[
\begin{align*}
(6) \quad U_1 d (t_g - t_{mo}) &= C \pi d_x \frac{t_m}{d} \\
(7) \quad C \pi d_x d_x &= U_2 \pi K \ d (t_{mi} - t) 
\end{align*}
\]
Since the heat transmitted through any given cylindrical layer of metal must be the same as that passing through any other layer, this heat must be independent of the cylindrical layer diameter, \( d_x \), and must be a function of \( L \) only; i.e.,

\[
(9) \ C\pi d_x \ dx = f(L)
\]

With the help of the last equation, the preceding three can be modified, as shown in the appendix, to the following forms:

\[
(6a) \ A_1 (t_g - t_{mc}) = A_2 (t_{mo} - t_{mi}) \\
(7a) \ A_2 (t_{mo} - t_{mi}) = A_3 (t_{mi} - t) \\
(8a) \ A_3 (t_{mi} - t) = f(L)
\]

where,

\[
A_1 = U_1 \pi d \\
A_2 = \frac{C\pi}{\log e} \frac{1}{k} \\
A_3 = U_2 \pi kd \\
f(L) = \frac{w \pi k^2 d^2}{4} v \frac{\partial t}{\partial L}
\]

If for direct comparison between the gas temperature, \( t_g \), and the water temperature, \( t \), the common form for the heat transmission equation, were used, we should have,

\[
(10) \ U_1 d (t_g - t) = f(L), \text{ or} \\
(10a) \ A (t_g - t) = f(L)
\]

where,

\[
A = U_1 d
\]
The relation between this "resultant" coefficient, $U$, and the "component" coefficients, $U_1, U_2, C$, is shown to be,

$$\frac{1}{A} = \frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3}$$

or similar to the expression for electrical resistances in series. Since $U, U_1, U_2, C$ increase with "heat conductance", their reciprocals, and therefore also, $\frac{1}{A}, \frac{1}{A_1}, \frac{1}{A_2}, \frac{1}{A_3}$ represent what may be called "heat resistances".

There is thus an analogy between the expressions for electrical resistances and for heat resistances in series, which is rather striking.

Rewriting equation (11) with proper substitutions, we get,

$$\frac{1}{U} \frac{d}{d} = \frac{1}{U_1} \frac{d}{d} + \frac{\ln e K}{C} + \frac{1}{U_2} \frac{d}{d} k$$

or

$$\frac{1}{U} = \frac{1}{U_1} + \frac{d \ln k}{C} + \frac{1}{KU}$$

Since,

$$d_i = k d,$$

the last equation may also be written as,

$$\frac{1}{U} = \frac{1}{U_1} + \frac{d_i \ln k}{K C} + \frac{1}{K V}$$

from which it is apparent that if, for a constant inside diameter, $d_i$ of pyrometer tube, the thickness of the latter and therefore also $k$ be varied, different values of $U$ will be ob-
tained. If now three such thicknesses were investigated, we should get three equations with three unknowns, \( U_1 \), \( U_2 \) and \( C \), and these equations would be independent of each other. By solving these equations, the values of \( U_1 \), \( U_2 \) and \( C \) would be found as separate coefficients rather than their resultant value, \( U \), which is the coefficient usually measured.

Then the value of \( U \) could be calculated, from eq. (13) for various thicknesses of tube, without the necessity of investigating each case experimentally. Also, knowing the effect of metal thickness upon the resultant coefficient of heat transmission, \( U \), design problems involving the thickness of pyrometer tubes could be analyzed and solved rationally.

Beside the mechanical problem of heat transmission involved in the electric furnace pyrometer design, there is the electrical problem of measuring and recording water temperature rise electrically.
For indicating purposes, copper-advance thermocouples may be used. Their calibration curve is shown in Fig. 1.

Figure 1.
To increase the thermal e. m. f. a number of them may be included in the same temperature plug and connected at their cold ends in series. Because of the small room occupied by these thin thermocouples, it would be no difficulty of placing as many as 10 to 20 of them in the same plug, and the temperature of water could be measured within. If now, one such plug is screwed into the cold end of the pyrometer tube and another one into the hot end of this tube, the difference between the two thermal e. m. f's. will be proportional to the difference of temperature between the hot and cold ends of the pyrometer tubes. This is evident if it is noted that the cold ends of the two thermocouples are at the same temperature, too. Then the emf's generated by the thermocouples are,

\[(14) \quad L_2 = K(t_2-t_0)\]

\[(15) \quad L_1 = K(t_1-t_0)\]

If now, the difference between these two e. m. f's is measured by means of a differential millivoltmeter, the reading of the latter in millivolts will be proportional to the temperature difference, \(t_2-t_1\). Thus,

\[(16) \quad L_2-L_1 = K(t_2-t_1)\]

The differential millivoltmeter may be calibrated to read, the difference between furnace and room temperature; or it may be calibrated to read o/o excess air directly.

Thus, for the first case, we combine equations (4a)
relation is given by equation (21a). It is to be noted also, that if in the case of theoretical air requirements, the assumption of 12,000 Btu/# coal has an error of say 10%, then since \(0.00008\) is about 10% of \(0.00075\), a 10% error in the former results in only 10% of 10% or 1% in the sum of \(0.00075 + 0.0008\ = 0.00083\). For 100% excess air this error is reduced to 1%, etc. Since the usual condition is 100% or more excess air, the approximation of \(\frac{1}{H}\) = 0.00008 is within permissible limits of error, and equation (21a) is justified.

Evidently then by combining equations (17) and (21a) we can get a direct relation between millivoltmeter readings, \(e, e_1\), and \(\%\) air, \(n\), used in the furnace. Thus we have,

\[
(22) \quad t_e - t_w = \frac{337 d_i \nu C_p (e_2 - e_1)}{100 (1+n) UL} = t_g - t_a = \frac{f}{\left( \frac{7.5n}{10,000} + 0.00008 \right)} C_p
\]

or

\[
(23) \quad n = \left( \frac{k_f (1+n) UL}{337 d_i \nu C_p (e_2 - e_1)} - 0.00008 \right) \frac{10,000}{7.5} = \frac{3.45 k_f (1+n) UL}{d_i \nu C_p} \frac{1}{e_2 - e_1} - 0.107 = \frac{k_2}{e_2 - e_1}
\]

where,

\[
k_2 = \frac{3.45 k_f (1+n) UL}{d_i \nu C_p}
\]

by means of the last equation, the millivoltmeter may evidently be calibrated to read \(\%\) air in furnace directly.

If the furnace temperature difference is to be recorder, the thermocouples are replaced by resistance coils of appreciable resistance temperature coefficient. Very thin enamelled copper wire is best for the low temperatures (below 200°F)
encountered in the water of the pyrometer tube. Thus, since,

\[
(24) \quad R_1 = R_0 (1 + (t_1 - 32))
\]

\[
(25) \quad R_2 = R_0 (1 + (t_2 - 32))
\]

where \( R_0 \) - resistance of the wire at 32°F we have,

\[
(26) \quad R_2 - R_1 = R_0 a (t_2 - t_1)
\]

Therefore, the water temperature rise may be calculated by measuring the difference of resistance between two copper coils placed at the hot and cold ends of the pyrometer tube, respectively. The relation between the resistance difference, \( R_2 - R_1 \), and the furnace temperature difference is the same as that given by eq. (17) for thermocouples, if \( K \) is replaced by \( R_0 a \). The same holds true for the \( \% \) air relation, as given by eq. (23).

To measure the difference of resistance, \( R_2 - R_1 \), automatically, the differential galvanometer method is used. The galvanometer will read zero when sufficient balancing resistance is added in series with the smaller resistance, \( R_1 \), to balance the larger resistance \( R_2 \), i.e., when the voltage across the first two resistances is equal to that across the last resistance. When due to change in temperature, the decrease or increase of resistances unbalances the voltages, the galvanometer will deflect in either direction depending upon the direction of unbalance.

If a valvanometer relay is used, similar to the one used for flow measurement by the Sargent Steam Meter Co.,
then the deflection of the galvanometer may be caused to act like an automatic switch closing the field circuit of a series motor of the Universal type, and the motor will rotate in a direction corresponding to that of the galvanometer deflection. The motor may be made to move a rheostat handle and thus vary the balancing resistance \( R = R_2 - R_1 \) until the galvanometer returns to its zero position, the field of the motor is automatically opened by the galvanometer relay, and the motor is stopped.

The balancing resistance \( R = R_2 - R_1 \) may be calibrated to read directly either furnace temperature difference, \( t_g - t_a \), or \( \% \) excess air, \( n - 1 \), as may be desired. The motor may be made to move a graphic pen as well as an indicating pointer, and thus the pyrometer may be made recording.

FURNACE PYROMETER CALIBRATION

The pyrometer described was tested on a 350 H. P. Sterling boiler at the Armour Institute. The water was supplied by a 3/4" pipe line at city water pressure, the flow being regulated by a valve. A monometer at the cold end of the pyrometer tube indicated the pressure of the flowing water at the beginning of the tube, and when at the hot end, the water was allowed to discharge freely into the sewer at atmospheric pressure without any restricting valves, the monometer measured directly the friction pressure drop through the pyrometer tube. By actually weighing the
water discharge, the monometer may be calibrated to read lbs. of water per minute, directly. The monometer calibration curve is shown in Fig. 2, below: Figure 2.
By setting the valve until the monometer indicated corresponded to the desired flow, the latter could be maintained constant during each run.

Specially made temperature cups were screwed into - Y - fittings, once at the cold and another at the hot end of the pyrometer tube. These cups were partially filled with oil, and calibrated mercury thermometers were immersed in this oil. These thermometers gave the cold and hot end temperatures of the water flowing through the pyrometer tubes.

For measuring the furnace temperatures at the different points of the bridge wall, a thermocouple twelve feet long was obtained and marked every foot of its length. Also a small fire door between the bridge wall and the first row of boiler tubes was opened and an asbestos board with an opening of the asbestos board, without the objection of letting a lot of cold air into the furnace.

The thermocouple was connected to a Leeds & Northrop potentiometer type of millivoltmeter, the thermal e. m. f. being balanced against the voltage drop across a variable calibrated resistance, produced by the e. m. f. of a standard cell. When the voltages were balanced, the thermal e. m. f. was disconnected, and the drop across the variable resistance was measured by the millivoltmeter. In that way, the error due to the voltage drops across the connecting wires were avoided.
The test consisted of adjusting the flow to a predetermined constant value by means of the valve and the monometer; of inserting the thermocouple to the last marked foot, and taking the temperature reading by means of the millivoltmeter; and finally, of reading the two mercury thermometers in the cold and hot end temperature cups.

The thermocouple would then be moved out to next foot mark, and the readings would be repeated. This was done for eight different foot marks, corresponding to almost every foot along the bridge wall.

A series of such tests were made on different days in order to cover as large a range of furnace temperature as possible.

A preliminary calibration test of the thermocouple was made in an electric furnace against a standard platinum thermocouple.

RESULTS AND CALCULATIONS.

The calibration curve of the 12 foot thermocouple is shown in Figure 3. As shown by the curve, the calibration was made up to 2200° F.
Table 1 gives a sample of date taken for the furnace temperature distribution and heat transmission tests. From the millivoltmeter readings and the thermocouple calibration curve, the furnace temperatures were figured out at different points of the bridge wall.
TABLE 1:

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<th>FURANCE</th>
<th>COLD</th>
<th>HOT</th>
<th>TEMP.</th>
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Because of the variation of average furnace temperatures during any one test, it was necessary to take the readings for the conditions as found and then tabulate them according to water temperature rises produces. Thus all readings corresponding to within $\pm 2^\circ F$ on either side of say $40^\circ F$ rise were tabulated together, and the furnace temperatures for the different points along the bridge wall for this average condition were thus available for comparison. Naturally, the tables for some such water temperature rises were more complete than for others, and special efforts were then made to secure the readings at the points and for the water temperatures for which one was short of data. The highest water temperature rises investigated were around the $90^\circ F$ point.

A sample of such tabulated data as well as the averages of the furnace and water temperatures is given in Table 2. The latter also includes the calculated $U$ for the particular furnace temperature difference under consideration.
TABLE 2.
It will be noted that for any given bridge will point of this table the furnace temperature varies as much as 5% above the average temperature at this point. This indicates the variation of the temperature distribution along the bridge wall with time. If the variations at all points along the bridge wall were all in the same direction or non-compensating, there would be a maximum possible, sampling error of 5%. Ordinarily there would be some compensation, so that the variation of the relation between the average furnace temperature and the average water temperature rise would be less than 5%. However, to play safe, we may state that as efficient as about 10% of tube along the bridge wall may be for sampling purposes as compared with that of point sampling, there is still a maximum possible error of 5% due to the variation of bridge wall temperature distribution.

It may now be in order to remark that the pyrometer tube was slightly covered with soot or slag, most of which was usually carried away by the moving gases. Several attempts were made to note the variation of the results, if the materials on the pyrometer tube were scraped off. It was found that there was about 1% increase in conductivity, when the tubes were thus treated. Considering the 5% possible error in the sampling efficiency of the tube, it is evident that the effect of soot accumulation outside the tube is negligible.

The average temperature distribution curves along the
bridge wall for several water temperature rises are plotted in Fig. 4 below.

Figure 4.
Figure 5 gives the U–curve of the values of $U$ calculated for different furnace temperature differences, from the tables given in the appendix and similar to table 2.

From the curve values of $U$ could be found for any furnace temperature differences, $t_g - t_a$, and used for calibrating the indicating or recording pyrometer to read furnace temperature differences directly.

After a comparison is made between these values of $U$ and those between these that will be obtained from the heat transmission test using an electric furnace, it is hoped that a rational relation between $U$ and overall temperature difference will be evolved.
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+ d: 35.56
- b: 14.57
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$V = 10.05$
Average $T = 16.74$

$T = 66.64$

$U = 10.27$
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Average $T = 1834$

$n = 63$

$U = 224$
Average $T = 1975$

$\theta = 19.15^\circ$

$V = 22.98$