CONCRETE FLAT SLAB DESIGN
ACCORDING TO CHICAGO,
BUILDING DEPARTMENT RULING

BY

E. W. CHAMBERLIN

ARMOUR INSTITUTE OF TECHNOLOGY
1915
Chamberlin, E. W.
Concrete flat slab design
according to Chicago
A THESIS
Presented by
Earl William Chamberlin
To The
PRESIDENT AND FACULTY
of
ARMOUR INSTITUTE OF TECHNOLOGY
For The Degree Of
CIVIL ENGINEER
In Accordance With Prescribed Requirements

Approved:

[Signatures]

May 7th, 1915.
CONCRETE FLAT SLAB DESIGN

ACCORDING TO CHICAGO BUILDING DEPARTMENT RULING.

The methods of design for concrete flat slab construction given in the textbooks on concrete are too long and complex to be satisfactory to the average engineer. Furthermore, the improbability of the theoretical conditions being found in actual construction makes the application of these formulas doubtful. A number of carefully conducted tests for actual stresses have been made on flat slab floors during the development of this type of construction, and it is from the information thus obtained that the empirical formulas now in general use by the flat slab designers have been developed.

For several years prior to 1914, no type of flat slab construction had the approval of the Chicago Building Department, until a satisfactory report of an actual test of the construction conducted by a recognized commission of engineers had been filed with the City Building Department. This report including recommended design formulas, when placed on file, became upon its acceptance the criterion for all later designs submitted by the company controlling the patents on the type tested. The result of this arrangement was that flat slab designs for buildings in Chicago could be submitted only by the companies exploiting the various patented types.
In answer to the urgent demand for uniform design standards for flat slab construction regardless of the patent situation, a Flat Slab Ruling was issued by the Building Commission in August 1914. This Ruling which will probably later become a part of the building ordinance, gives formulas for slab thickness, column capital diameter and for bending moments, thus putting designers on the same footing as far as flat slab buildings in Chicago are concerned.

The Ruling, a copy of which is attached hereto, provides for flat slabs with both two way and form way reinforcing. The calculations for concrete sizes and reinforcement for a typical four-way flat slab are given below. Authority for the formulas used in this calculation will be found in the Ruling or in the standard text books.

Assume as a typical example a 20' x 20' interior panel with a 200 pound live load. Reinforcement to be of four-way type and the design according to Chicago Building Department Ruling.

**Slab.**

Assuming an 8" slab \( t = 0.023 \times \sqrt[3]{W} = 0.023 \times 20 \sqrt[3]{300} = 7.96" \.

One thirty second of the span = \( \frac{20 \times 12}{32} = 7.5" \) or less than the thickness required by the formula. Use 8" slab.

**Drop Panel.**

If all reinforcing rods are assumed to be two or more panels in length as is customary and the ends lapped for bond only, the depth of drop will depend on the amount of steel in one diagonal and one rectangular band, or upon the punching shear on the perimeter of the cap. If 1/2" rods are used and a minimum covering of 1/2" concrete required, the effective depth
in the drop panel will be 1 1/2" less than the total drop thickness. The total depth of the slab being determined as above, the amount of steel required in one diagonal band depends on the effective depth of the slab and the moment, \( WL^2 \). The amount of steel in one rectangular band depends upon the same effective depth and a moment of \( WL^2 \) and is therefore equal to \( \frac{60}{10} \) or 1 1/2 times the steel in the diagonal band. According to the Ruling the negative moment at the capital equals \( WL^2 \) and is resisted by the steel in one diagonal and one rectangular band, or an amount equal to 2 1/2 times one diagonal band. Therefore if rods are lapped for band only, the effective depth of the drop varies from the effective depth of the slab directly as the moment and inversely as the the steel for one diagonal band.

In this example the effective depth of the slab = 8-1 or 7". The effective depth required at the drop therefore equals 7 x \( \frac{60}{15} \) = 7 x \( \frac{8}{5} \) = 11.2" and the total depth = 11.2 + 1.5 = 12.7", say 12".

The width of the drop will be determined by the compression on it figured as a beam or by the punching shear along its perimeter. For ordinary loads however, the width is determined by the compression, applying the regular beam formula, width of drop = \( \frac{M}{Kd} = \frac{WL^2}{15 Kd} = \frac{3000 \times 20^2 \times 12}{15 \times 113 \times 11\frac{1}{4}} = 6.7' = .335L \) for interior panels. For wall panels supported by columns and girders the negative moment is \( \frac{WL^2}{12} \) and the width of drop required would be \( \frac{1}{12} \) of the value just found. As it would be impracticable to use different sizes for interior and wall panel drops, an average width of .35L for all is suggested, giving for this example \( .35 \times 20 = 7'0" \)

With the concrete dimensions established as above the
reinforcement is easily calculated by the formulas for bending moment given in the Ruling. The moment resisted by one diagonal band = $\frac{WL}{60}$ and the area of steel required is by the regular beam formula,

$$\text{Asd} = \frac{M}{\text{jdfs}} = \frac{\frac{WL^2}{60}}{\text{jdfs}} = \frac{WL^2 x 12}{60 x 0.87 x 1800} = \frac{WL^2}{78200d} = \frac{3000 \times 20}{78200 \times 7} = 1.96 \text{ square inches of steel. Ten one half inch round rods would make this area but the maximum allowable spacing of 9" will require that fourteen 7/16 inch round rods be used.}

The moment resisted by one rectangular band = $\frac{WL}{40}$ or $1\frac{1}{2}$ times the moment resisted by one diagonal band. The amount of steel required for one rectangular band therefore, equals $1\frac{1}{2} \times 1.96 = 2.96$ or $15, \frac{1}{2}$-inch rods. With the depth of drop established as above, the reinforcement thus determined will provide for the negative moment over the column. For walls supported by columns and girder the moments of $\frac{WL^2}{60}$ and $\frac{WL^2}{40}$ become $\frac{WL^2}{50}$ and $\frac{WL^2}{33}$ respectively. The areas of steel found for one diagonal and one rectangular band for interior panels will therefore be multiplied by the ratios $\frac{60}{50}$ and $\frac{40}{30}$ respectively to determine the reinforcement in wall panels.

Briefly summarized, the design of four-way flat slabs according to Chicago Ruling where rods are lapped for bond only, is as follows:

- Diameter of column cap = $0.225L$
- Slab thickness = $t = 0.023 \times \frac{LVW}{LW}$
- Depth of drop panel = $8 \times (t-1) + 1\frac{1}{2}"$
- Length of side of drop panel, say $0.35L$
- Area of steel in one diagonal band = $\text{Asd} = \frac{WL^2}{78200d}$
- Area of steel in one rectangular band = $\text{Asd} \times \frac{60}{50}$ and area of steel in one rectangular band = $\text{Asd} \times \frac{1\frac{1}{2}}{33}$

The attached blue print shows concrete sizes for various spans and live loads derived as above.
FLAT SLAB DIMENSIONS.

Depth \( T \) according to Chicago Ordinance for all flat slabs. Depths \( H \) according to Chicago Ord. for four-way reinforcement lapped for bend only.

<table>
<thead>
<tr>
<th>( L )</th>
<th>( C )</th>
<th>( D )</th>
<th>100° LL</th>
<th>150° LL</th>
<th>200° LL</th>
<th>250° LL</th>
<th>300° LL</th>
<th>400° LL</th>
</tr>
</thead>
<tbody>
<tr>
<td>16-0</td>
<td>5-8°</td>
<td>3-8&quot;</td>
<td>6&quot;</td>
<td>9\frac{1}{2}&quot;</td>
<td>6&quot;</td>
<td>9\frac{1}{2}&quot;</td>
<td>6\frac{1}{2}&quot;</td>
<td>10&quot;</td>
</tr>
<tr>
<td>17-0</td>
<td>6-0°</td>
<td>3-10</td>
<td>6&quot;</td>
<td>9\frac{1}{2}&quot;</td>
<td>6\frac{1}{2}&quot;</td>
<td>10\frac{1}{4}&quot;</td>
<td>7\frac{1}{2}&quot;</td>
<td>11\frac{1}{2}&quot;</td>
</tr>
<tr>
<td>18-0</td>
<td>6-4&quot;</td>
<td>4-0&quot;</td>
<td>6&quot;</td>
<td>9\frac{1}{2}&quot;</td>
<td>6\frac{3}{4}&quot;</td>
<td>10\frac{3}{4}&quot;</td>
<td>7&quot;</td>
<td>11&quot;</td>
</tr>
<tr>
<td>19-0</td>
<td>6-8&quot;</td>
<td>4-4&quot;</td>
<td>6&quot;</td>
<td>9\frac{1}{2}&quot;</td>
<td>7\frac{1}{4}&quot;</td>
<td>11\frac{1}{2}&quot;</td>
<td>7\frac{1}{4}&quot;</td>
<td>11\frac{1}{2}&quot;</td>
</tr>
<tr>
<td>20-0</td>
<td>7-0&quot;</td>
<td>4-6&quot;</td>
<td>6&quot;</td>
<td>9\frac{1}{2}&quot;</td>
<td>7\frac{1}{2}&quot;</td>
<td>12&quot;</td>
<td>7\frac{1}{2}&quot;</td>
<td>12&quot;</td>
</tr>
<tr>
<td>21-0</td>
<td>7-4&quot;</td>
<td>4-8&quot;</td>
<td>6\frac{1}{4}&quot;</td>
<td>10&quot;</td>
<td>8&quot;</td>
<td>12\frac{3}{4}&quot;</td>
<td>8&quot;</td>
<td>12\frac{3}{4}&quot;</td>
</tr>
<tr>
<td>22-0</td>
<td>7-8&quot;</td>
<td>5-0&quot;</td>
<td>6\frac{1}{2}&quot;</td>
<td>10\frac{1}{2}&quot;</td>
<td>8\frac{1}{2}&quot;</td>
<td>13\frac{1}{4}&quot;</td>
<td>8\frac{1}{2}&quot;</td>
<td>13\frac{1}{4}&quot;</td>
</tr>
<tr>
<td>23-0</td>
<td>8-1&quot;</td>
<td>5-2&quot;</td>
<td>7&quot;</td>
<td>11&quot;</td>
<td>8\frac{1}{2}&quot;</td>
<td>13\frac{1}{4}&quot;</td>
<td>8\frac{1}{2}&quot;</td>
<td>13\frac{1}{4}&quot;</td>
</tr>
<tr>
<td>24-0</td>
<td>8-5&quot;</td>
<td>5-4&quot;</td>
<td>7\frac{1}{2}&quot;</td>
<td>11\frac{1}{2}&quot;</td>
<td>9&quot;</td>
<td>14\frac{1}{4}&quot;</td>
<td>9&quot;</td>
<td>14\frac{1}{4}&quot;</td>
</tr>
<tr>
<td>25-0</td>
<td>8-9&quot;</td>
<td>5-8&quot;</td>
<td>7\frac{1}{2}&quot;</td>
<td>12&quot;</td>
<td>9\frac{1}{2}&quot;</td>
<td>15&quot;</td>
<td>9\frac{1}{2}&quot;</td>
<td>15&quot;</td>
</tr>
<tr>
<td>26-0</td>
<td>9-1&quot;</td>
<td>5-10&quot;</td>
<td>8&quot;</td>
<td>12\frac{3}{4}&quot;</td>
<td>9\frac{1}{2}&quot;</td>
<td>15\frac{1}{2}&quot;</td>
<td>9\frac{1}{2}&quot;</td>
<td>15\frac{1}{2}&quot;</td>
</tr>
</tbody>
</table>

Note: Dimension \( C \) is shown uniformly as \( 0.35L \).

(For interior pbl. \( C = 0.0043 \times \frac{W \times L^3}{(H/18)^3} \) For wall panel with wall cols. \( C = 0.0053 \times \frac{W \times L^3}{(H/18)^3} \) \( C \) in inches)
RULING

On The Design Of

REINFORCED CONCRETE FLAT SLABS

In The City Of Chicago


Issued By

Henry Ericsson, Commissioner of Buildings


Aug. 18, 1914
THE DESIGN OF FLAT SLABS SHALL BE IN ACCORDANCE
WITH THE FOLLOWING RULING.

Definitions.

Flat slabs as understood by this ruling are reinforced con-
crete slabs supported directly on reinforced columns with or
without plates or capitals at the top, the whole construction be-
ing hingeless and monolithic without any visible beams or girders/
the construction may be such as to admit the use of hollow
panels in the ceiling or smooth ceiling with depressed panels in
the floor.

The column capital shall be defined as the gradual flaring out
of the top of the column without any marked offset.

The drop panel shall be defined as a square or rectangular de-
pression around the column capital extending below the slab ad-
jacent to it.

The panel length shall be defined as the distance center to
center of columns of the side of a square panel, or the average
distance center to center of columns of the long and short sides
of a rectangular panel.

COLUMNS.

The least dimensions of any concrete column shall be not less
than one twelfth \((1/12)\) the panel length, or one-twelfth \((1/12)\)
the clear height of the column.

SLAB THICKNESS.

The minimum total thickness of the slab in inches shall be
determined by the formula:-

\[
t = 0.023 L \times \sqrt{w}
\]

where \(t\) is the total thickness of slab in inches.

\(L\) is panel length in feet.

\(w\) is total live and dead load in lbs. per square foot.

In no case shall the slab thickness be less than one thirty-
secondth (1/32) of the panel length for floors, and one-fortieth (1/40) of the panel length for roofs, and also not less than six inches (6"").

COLUMN CAPITAL.

The diameter of the column capital shall be measured where its vertical thickness is at least one and one-half inches (1½") and shall be at least two hundred and twenty-five thousandths (0.225) of the panel length.

The slope of the column capital shall nowhere make an angle with the vertical of more than forty-five degrees. Special attention shall be given to the design of the column capital in considering eccentric loads, and the effect of wind upon the structure.

DROP PANEL.

The depth of the drop panel shall be determined by computing it as a beam, using the negative bending moment specified elsewhere in this ruling. The width and length shall be determined by the allowable unit shearing stresses on the perimeter, given below.

SHEARING STRESSES.

The allowable unit punching shear on the perimeter of the column capital shall be three-fifteenth (3/50) of the ultimate compressive strength of the concrete as given in section 546 of the building ordinance. The allowable unit shear on the perimeter of the drop panel shall be three one-hundredths (3/100) of the ultimate compressive strength of the concrete. In computing shearing stress for the purpose of determining the resistance to diagonal tension the method specified by the ordinance shall be used.

PANEL STRIPS.

For the purpose of establishing the bending moments and the resisting moments of a square panel, the panel shall be divided into strips known as strip A and strip B. Strip A shall include the reinforcement and slab in width extending from the center line of the
column for a distance each side of this center line equal to one-quarter (1/4) of the panel length. Strip B shall include the reinforcement and slab in the half width remaining in the center of the panel. At right angles to these strips, the panel shall be divided into similar strips A and B, having the same widths and relations to the center line of the columns as the above strips. These strips shall be for designing purposes only, and are not intended as the boundary lines of any bands of steel used.

These strips shall apply to the system of reinforcement in which the reinforcing bars are placed parallel and at right angles to the center line of the columns, hereinafter known as the two-way system, and also to the system of reinforcement in which the reinforcing bars are placed parallel, at right angles to and diagonal to the center line of the columns hereinafter known as the four-way system.

**BENDING MOMENT COEFFICIENTS, INTERIOR PANEL, TWO-WAY SYSTEM.**

The negative bending moment taken at a cross-section of each strip A at the edge of a column capital or over it, shall be taken as WL/15. The positive bending moment taken at a cross-section of each strip A, midway between column centers shall be taken as WL/30. The positive bending moment taken at a cross-section of each strip B in the middle of the panel shall be taken at WL/60. The negative bending moment taken at a cross-section of each strip B on the center line of the column shall be taken at WL/60. In the formulas hereinabove give n

\[ W = \text{total live and dead load per lineal foot of each strip,} \]

\[ L = \text{panel length in feet.} \]

**BENDING MOMENT COEFFICIENTS, INTERIOR PANELS, FOUR-WAY SYSTEM.**

The negative bending moment taken at a cross-section of each strip A at the edge of the column capital or over it, shall be taken as WL/15. The positive bending moment taken at a cross-section of
each strip A, midway between column centers shall be taken as WL/40. The positive bending moment taken at a cross-section of each strip B in the middle of the panel shall be taken at WL/60. The negative bending moment taken at a cross-section of each strip B on the center line of the columns be taken at WL/60.

**BENDING MOMENT COEFFICIENTS, WALL PANELS.**

Whenever the coefficients 1/15, 1/30, 1/40, or 1/60 appear in the moments given for interior panels in either the two-way or the four-way systems, the coefficient 1/12, 1/25, 1/33, and 1/50 respectively shall be used in the moments for wall panels supported on concrete columns and girders.

When brick walls are used partly to support wall panels, these walls shall be stiffened by pilasters or piers as directed by the Commissioner of Buildings. Whenever the coefficients 1/15, 1/30, 1/40 or 1/60 appear in the moments given for interior panels in either the two-way or the four-way systems, the coefficients 1/10, 1/20, 1/27 and 1/40 respectively shall be used in the moments for such panels resting on brick walls.

**POINT OF INFLECTION.**

For the purpose of making the calculations of the bending moment at the sections away from the column capital, the point of inflection shall be considered as being one-quarter (1/4) the distance center to center of columns, both cross-wise and diagonally, from the center of the column.

**TENSILE STRESS IN STEEL AND COMPRESSIVE STRESS IN CONCRETE.**

The tensile stress in steel and the compressive stress in the concrete to resist the bending moment shall be calculated on the basis of the reinforcement and slab in the width included in a given strip, and according to the assumptions and requirements given in sections 545 to 548 inclusive of the building ordinance.

The steel shall be considered as being concentrated at the cen-
centrated at the center of gravity of all the bands of steel in a given strip.

For the four-way system of reinforcement the amount of steel to resist the negative bending moment over the support kn each strip A shall be taken as the sum of the areas of steel in one crossband and one diagonal band. The amount of steel to resist the positive bending moment of each strip B shall be considered as the area of the steel in a diagonal band. The amount of steel to resist the positive bending moment in each strip A shall be considered as the area of the steel in a cross-band, and the amount of steel to resist the negative moment in each strip B shall be the steel included in the width of strip B.

For the two-way system of reinforcement the amount of steel to resist the bending moment in any strip shall be considered as the area of steel included in the width of the strip.

In both systems of reinforcement the compressive stress in the concrete in any strip shall be calculated by taking the area of steel considered for each strip, and applying it in a beam formula based on the principles of section 548 of the building ordinance.

RECTANGULAR PANELS.

When the length of a panel does not exceed the breadth by more than five per cent (5%), all computations shall be made on the basis of a square with sides equal to the mean of the length and breadth. In no rectangular panel shall the length exceed four-thirds (4/3) the breadth.

For panels with length more than five per cent (5%) in excess of the breadth, the slab shall first be designed for a bending moment based on an assumed square panel with sides equal to the mean of the length and breadth of the rectangular panel.

For the four-way system of reinforcement the amount of steel found for the positive moment of each strip B by designing in this manner
shall be that used in the diagonal band. For the positive moment in each strip A, the required amount of steel in the cross-band shall be obtained by multiplying the steel used in the design of the assumed square panel by the cube of the ratio found by dividing the length or breadth of the rectangular panel by the side of the assumed square panel, for the long and short sides of the panel respectively. The compressive stresses shall be calculated on the basis of a width equal to one-half \((1/2)\) of the side of the assumed square panel, and on the assumptions used in the calculations of compressive stresses in square panels. In no case shall the amount of steel in the short side be less than two-thirds \((2/3)\) of that required for the long side.

For the two-way system of reinforcement, the amount of steel found for the positive and negative moment of each strip B by designing in this manner shall be obtained by multiplying the steel used in the design of the assumed square panel by the cube of the ratio found by dividing the length or breadth of the rectangular panel by the side of the assumed square panel, for the short and long side of the panel respectively. The method of obtaining the amount of steel required for each strip A, shall be the same as that given above for the four-way system.

WALLS AND OPENINGS.

Girders or beams shall be constructed under walls, and around openings, and to carry concentrated loads.

COMPUTATIONS.

Complete computations of interior and wall panels and such other portions of the building as may be required by the Commissioner of Buildings shall be left in the office of the Commissioner of Buildings when plans are presented for approval.

PLACING OF STEEL.

In order that the slab bars shall be maintained in the pos-
tion shown in the design during the work of pouring the slab, spacers and supports shall be provided satisfactory to the Commissioner of Buildings. All bars shall be secured in place at intersections by wire or other metal fastenings. In no case shall the spacing of bars exceed nine inches (9""). The steel to resist the negative moment in each strip B shall extend one-quarter (¼) of the panel length beyond the center line of the columns in both directions.

All splices in bars shall be made over the column head. The length of the splice beyond the center line of the column in both directions shall be at least two feet (2'), nor less than that necessary for the full development of the strength of the bar as limited by the unit bond stresses given by the ordinance. The splicing of adjacent bars shall be avoided as far as possible.

Slab bars which are lapped over the column, the sectional area of both being included in the calculations for negative moment, shall extend not less than twenty-five one-hundredths (25) of the panel length for cross-bands, and thirty-five one-hundredths (.35) of the panel length for diagonal bands, beyond the column center.

TEST OF WORKMANSHIP.

The Commissioner of Buildings or his representative may choose any two adjacent panels in the building for the purpose of ascertaining the character of workmanship. The test shall not be made sooner than the time required for the cement to set thoroughly, nor less than six weeks after the concrete had been poured.

All deflections under test load shall be taken at the center of the slab, and shall be measured from the normal unloaded position of the slab. The two panels selected shall be uniformly loaded over their entire area with a load equal to the dead load plus twice the live load, thus obtaining twice the total design load.
The load shall remain in place not less than twenty-four (24) hours. If the total deflection in the center of the panel under the test load does not exceed one eight-hundredth (1/800) of the panel length, the slab may be placarded to carry the full design live load. If it exceeds this amount of deflection, and recovers not less than eighty per cent (80%) of the total deflection within seven days after the load is removed, the slab may be placarded to carry the full design live load. If the deflection exceeds the allowable amount above specified, and the recovery is less than eighty per cent (80%) in seven days after the removal of the test load, other tests shall be made on the same or other panels, the results of which will determine the amount of live load the slabs will be permitted to carry.

GENERAL.

The design and execution of the work shall conform to the provisions of the Chicago building ordinances, and to correct principles of construction.

HENRY ERICSSON,
Commissioner of Buildings.

J.N.J.
Aug. 18th, 1914.